

**Transport Properties of
3D Topological Insulators and
a Candidate for Magnetic Weyl Semimetals**

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Chinese Academy of Sciences*

<http://nano.iphy.ac.cn/N08>

ICAM Summer School, Beijing, July 21, 2015

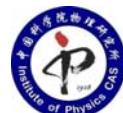
**Part I: Localization phenomena in
three-dimensional topological insulators**

- I. Introduction to localization
- II. Gate voltage tuning of chemical potential in TI
- III. Weak antilocalization in TI (perpendicular fields)
- IV. Weak antilocalization in TI (parallel fields)
- V. Electron-electron interaction effects
- VI. Crossover from 3D to 2D & observation of
Anderson localization
- VII. Summary and outlook

J. Chen et al., **PRL** 105, 176602 (2010); **PRB** 83, 241304 (R) (2011);
C. J. Lin et al., **PRB** 88, 041307 (R) (2013); J. Liao et al., **PRL** 114, 216601 (2015).

Collaborators

- Jun Chen, Tong Guan, Jian Liao, Chaojing Lin, Wenmin Yang
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- Huajun Chen, Xiaoyue He, Guanhua Zhang, Shuo Yang, **Kehui Wu**
Surface Physics Lab, IOP-CAS
- Yunbo Ou, Xiao Feng, **Ke He**, Xucun Ma, **Qi-Kun Xue**
Tsinghua University



Funding support: NSF-China, MOST 973 & Chinese Academy of Sciences

Acknowledgements (cont'd):

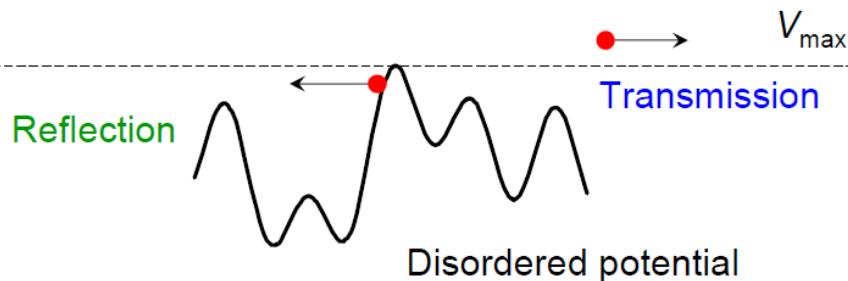
- Theoretical support:
Junren Shi & Xin-Cheng Xie, ICQM, Peking University
Alexander Mirlin, University of Karlsruhe
Xi Dai & Zhong Fang, IOP-CAS
- Experimental support:
Li Lv et al., IOP-CAS
Jurgen Smet & K. von Klitzing, MPI-Stuttgart

I. Introduction to localization



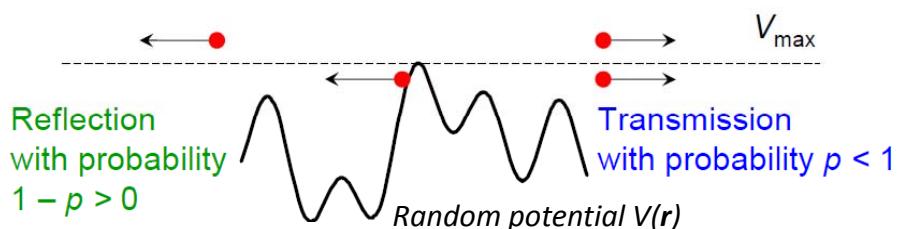
Increasing Disorder

Transport of a Classical Particle

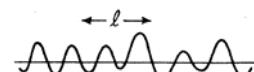


No prorogation occurs when particle's kinetic energy E_k is lower than V_{\max}

Quantum Particles ~ Waves

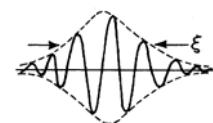


Diffusion \Leftrightarrow extended waves



vs.

Absence of transport \Leftrightarrow localized waves

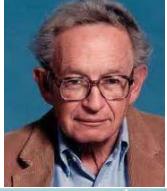


Anderson Localization

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

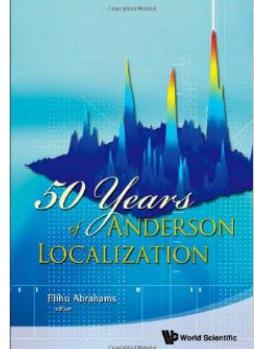
Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)



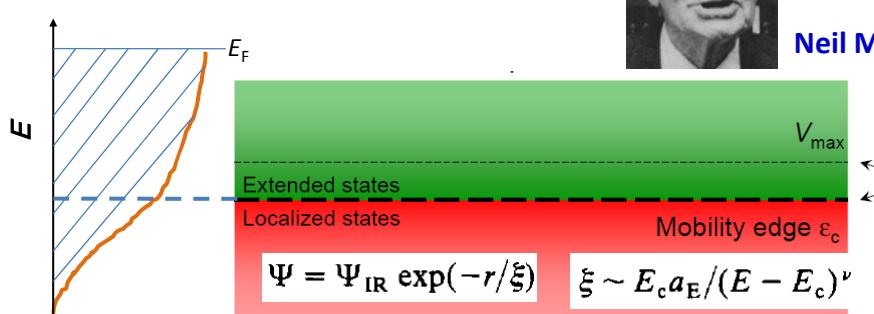
**1) light, microwave, acoustic wave ...
Classical waves**

**2) Electrons, spin wave, cold atoms ...
Quantum waves
Complicated by interactions**



Mobility Edge





$\Psi = \Psi_{IR} \exp(-r/\xi)$ $\xi \sim E_c a_E / (E - E_c)^\nu$

Variable range hopping: $\sigma \propto \exp\left[-\left(\frac{T_0}{T}\right)^{\frac{1}{d+1}}\right]$ $d=2,3$

N. F. Mott, *Adv. Phys.* **16**, 49 (1967)
N. F. Mott & E. A. Davis, *Electronic Processes in Non-Crystalline Materials* (1979, Oxford)

1977 Nobel Prize



Philip W. Anderson



Nevill F. Mott



John H. Van Vleck

Official press release: for their fundamental theoretical investigations of the electronic structure of magnetic and disordered systems.

Scaling Theory of electron localization

VOLUME 42, NUMBER 10

PHYSICAL REVIEW LETTERS

5 MARCH 1979

Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions

E. Abrahams

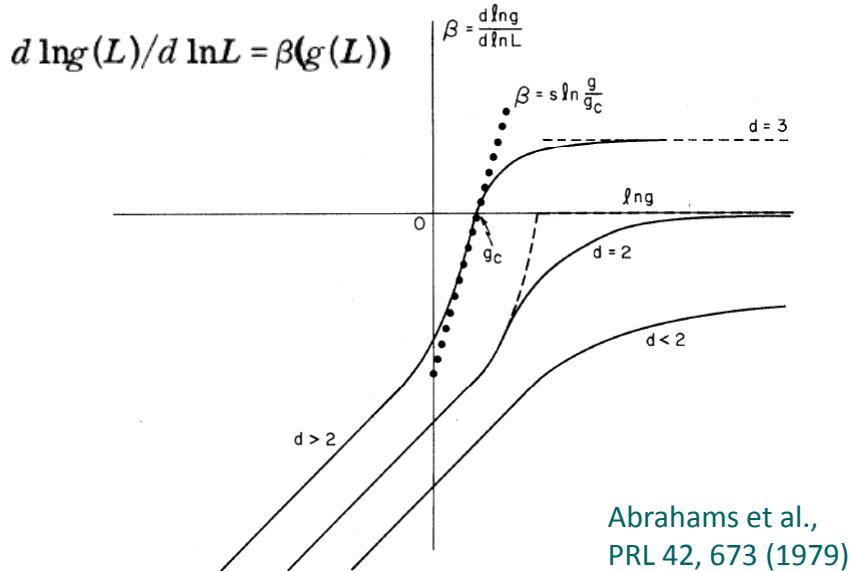
Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08854

and

P. W. Anderson,^(a) D. C. Licciardello, and T. V. Ramakrishnan^(b)*Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08540*
(Received 7 December 1978)

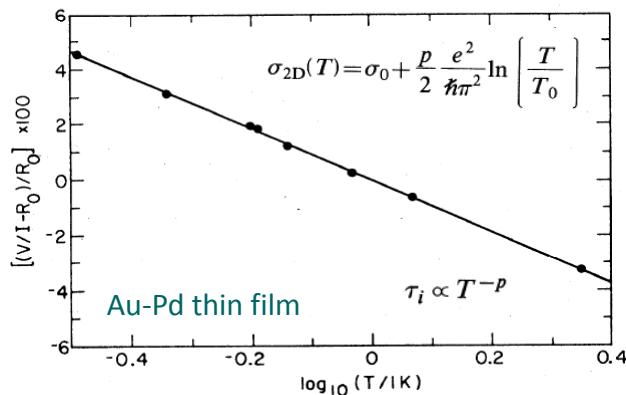
Arguments are presented that the $T=0$ conductance G of a disordered electronic system depends on its length scale L in a universal manner. Asymptotic forms are obtained for the scaling function $\beta(G) = d\ln G / d\ln L$, valid for both $G \ll G_c \sim e^2/h$ and $G \gg G_c$. In three dimensions, G_c is an unstable fixed point. In two dimensions, there is no true metallic behavior; the conductance crosses over smoothly from logarithmic or slower to exponential decrease with L .

Scaling Theory: No true metallic states in 2D



Weak Localization in 2D

$$(d \ln g / d \ln L) = -a/g \quad \Rightarrow \quad g(L) = g_0 - \frac{e^2}{\hbar \pi^2} \ln \left(\frac{L}{l} \right)$$



Dolan & Osheroff, PRL (1979)

Weak Localization: a precursor to Anderson Localization

At low T,

$$l_\phi \gg l_e$$

Without spin-orbit coupling:

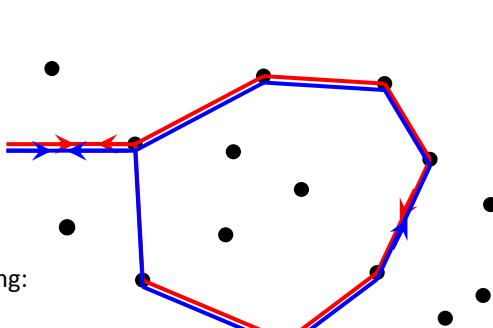
$$\rho_{\text{CW}} = \rho_{\text{CCW}}$$

Quantum
backscattering probability:

$$|A_{\text{CW}} + A_{\text{CCW}}|^2 = 4A_0^2$$

Classical
backscattering probability:

$$A_{\text{CW}}^2 + A_{\text{CCW}}^2 = 2A_0^2$$

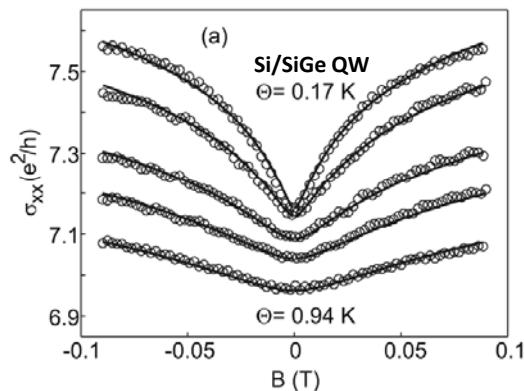


Consequence of Weak Localization

Temperature dependence: $\Delta\sigma^{\text{WL}} \sim \ln T$

Magnetic field dependence:

$$\Delta\sigma^{\text{WL}}(B) = \frac{e^2}{2\pi^2\hbar} \left[\psi\left(\frac{1}{2} + \frac{B_\phi}{B}\right) - \psi\left(\frac{1}{2} + \frac{B_e}{B}\right) + \ln\left(\frac{B_e}{B_\phi}\right) \right]$$

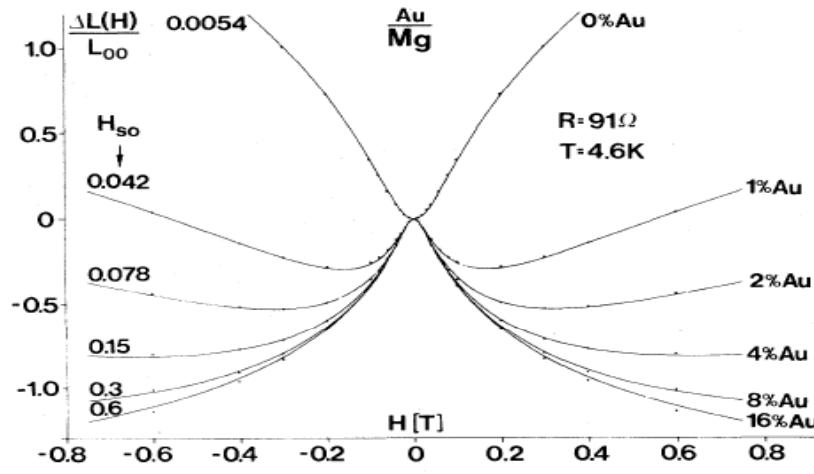


B. L. Altshuler et al.,
PRB 22, 5142 (1980)

T. Heizel,
Mesoscopic Electronics in Solid State Nanostructures,
p229 (Wiley VCH, 2007)

Spin-Orbit Coupling:

Crossover from weak localization to weak antilocalization



G. Bergman,
PRL 48, 1046 (1982)

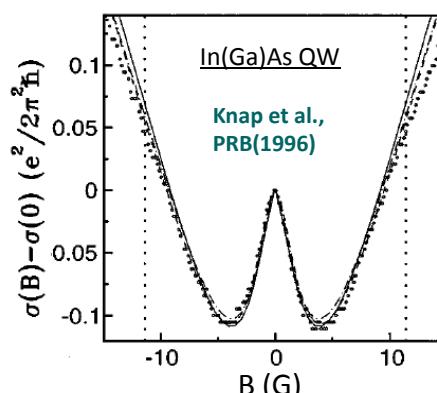
Hikami-Larkin-Nagaoka theory

$$\Delta\sigma(B) = -\frac{e^2}{\pi h} \left[\Psi\left(\frac{1}{2} + \frac{B_e}{B}\right) - \frac{3}{2} \Psi\left(\frac{1}{2} + \frac{B_i + 4/3B_{SO}}{B}\right) + \frac{1}{2} \Psi\left(\frac{1}{2} + \frac{B_i}{B}\right) \right]$$

B_e : elastic scattering
 B_i : inelastic scattering
 B_{SO} : spin-orbital coupling

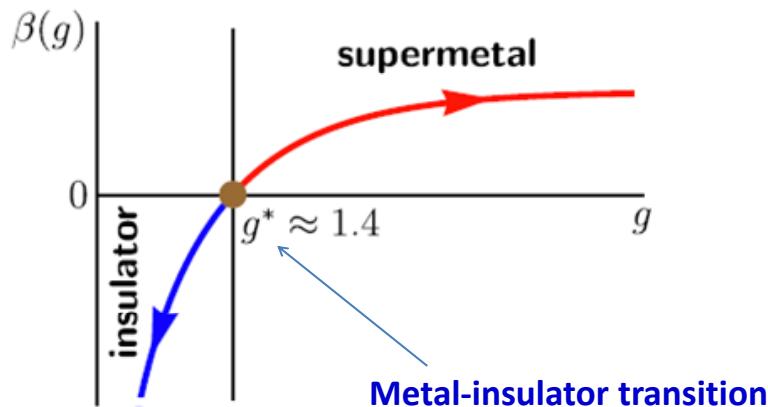
$$B_\beta = \frac{\hbar}{4D\tau_\beta}$$

D : diffusion constant
 τ : scattering time



Hikami et al., Prog. Theor. Phys. 63, 707 (1980)
 Bergmann, Phys. Rep. 107, 1 (1984)

Non-interacting 2D electron system with SOC



VOLUME 44, NUMBER 19

PHYSICAL REVIEW LETTERS

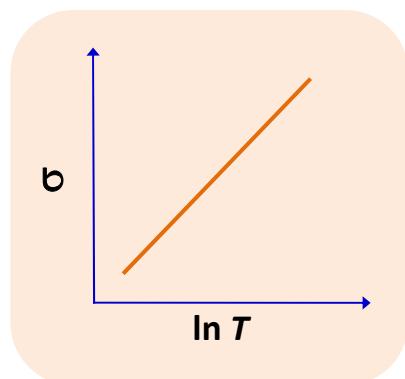
12 MAY 1980

Interaction Effects in Disordered Fermi Systems in Two Dimensions

B. L. Altshuler and A. G. Aronov
Leningrad Nuclear Physics Institute, Gatchina, Leningrad 188 350, U.S.S.R.

and

P. A. Lee
Bell Laboratories, Murray Hill, New Jersey 07974
 (Received 11 February 1980)

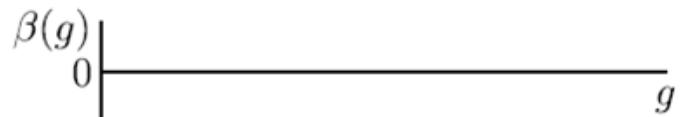


e-e interaction effects:

$$\delta\sigma = \frac{e^2}{\hbar} \frac{1}{4\pi^2} \left(2 - \frac{3}{2}\tilde{F}_\sigma\right) \ln(T\tau)$$

Review: Lee & Ramakrishnan,
Rev. Mod. Phys. **57**, 287 (1985)

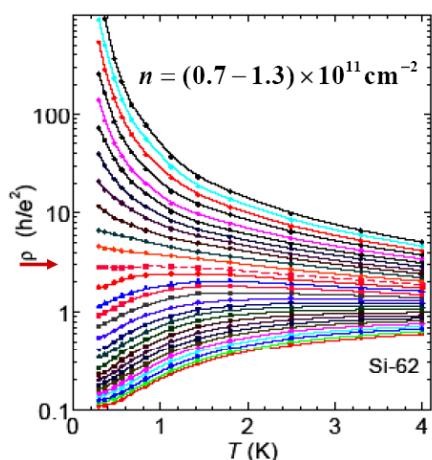
Interacting 2D electron system with SOC



insulator



Strongly interacting 2D electron systems



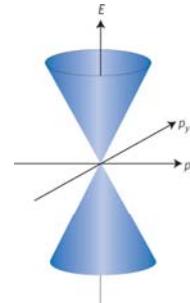
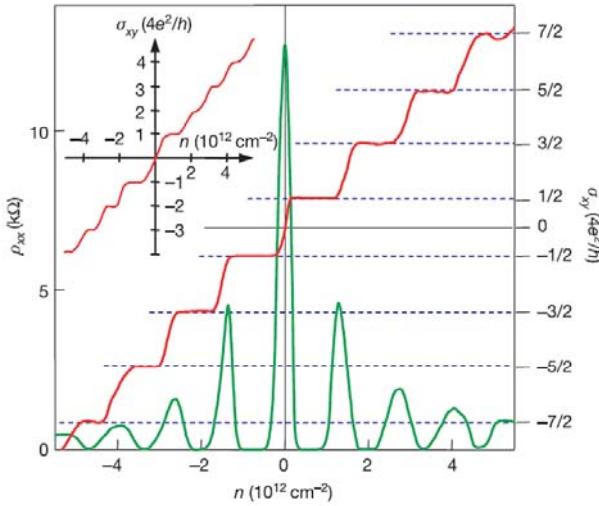
Low disorder
Low density $\Rightarrow r_s \gg 1$
 $r_s \equiv 1 / \sqrt{\pi n(a_B)^2}$

Metal-insulator transition



Kravchenko et al. Phys. Rev. B **50**, 8039 (1994)
Spivak et al., Rev. Mod. Phys. **82**, 1743 (2010)

2D Dirac electron systems



π Berry's phase

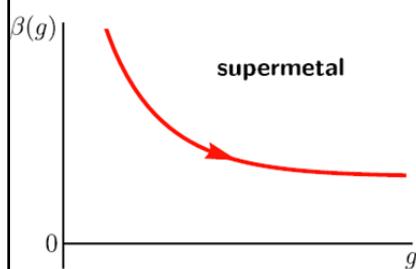
Also leading to
weak antilocalization
(WAL)

T. Ando et al.,
J. Phys. Soc. Jpn.
67, 2857 (1998)

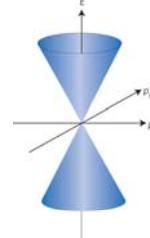
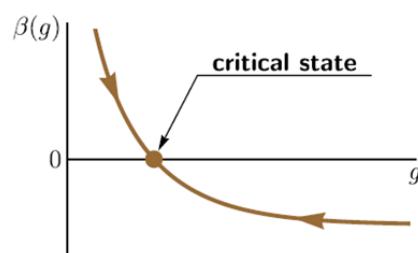
Novoselov & Geim et al., *Nature* **438**, 197 (2005)
Also reported by Y. B. Zhang et al., *Nature* **438**, 201 (2005)

Scaling behavior in 2D Dirac Fermion Systems

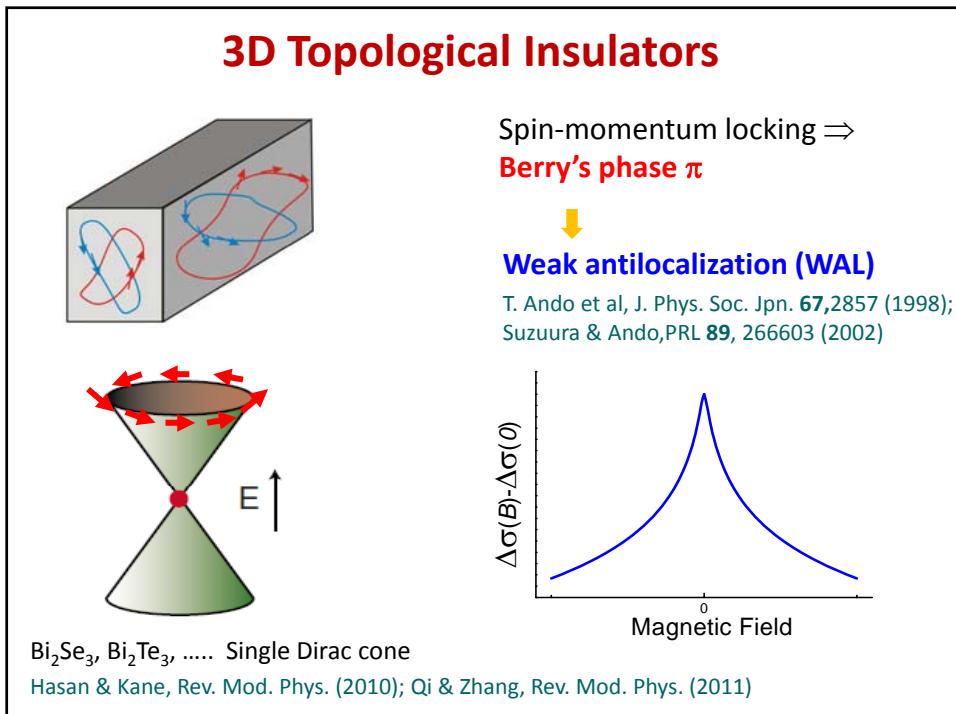
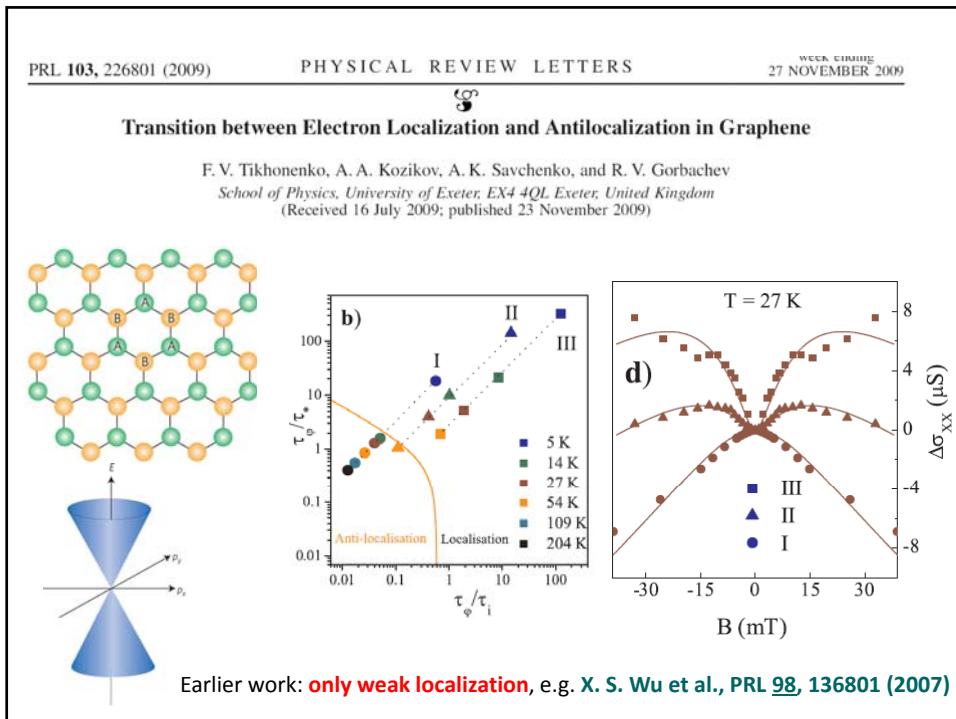
Non-interacting

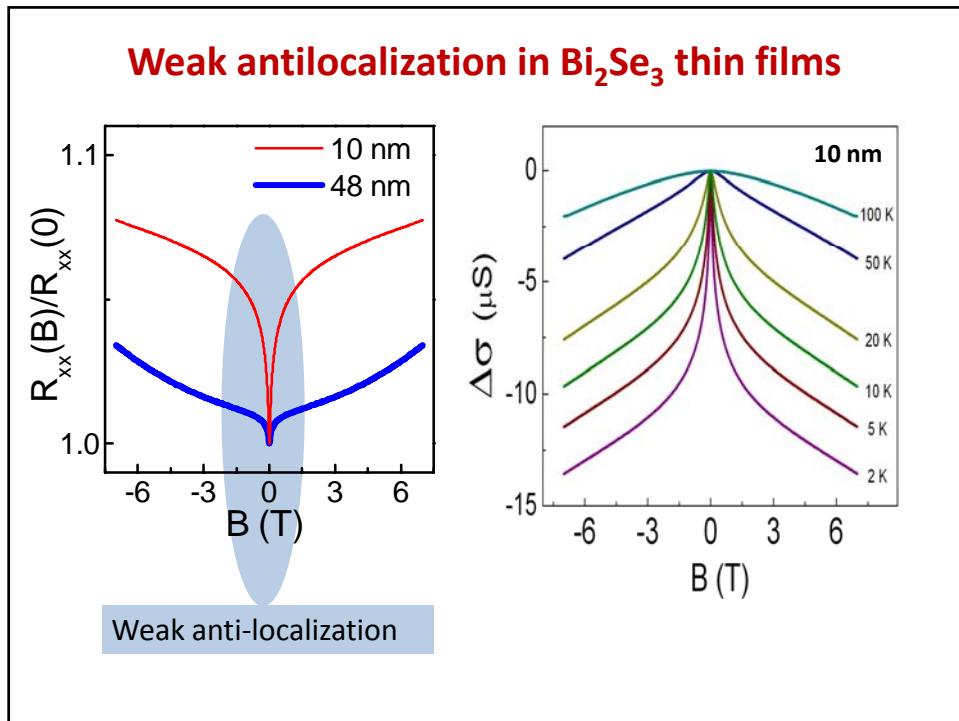


Interacting

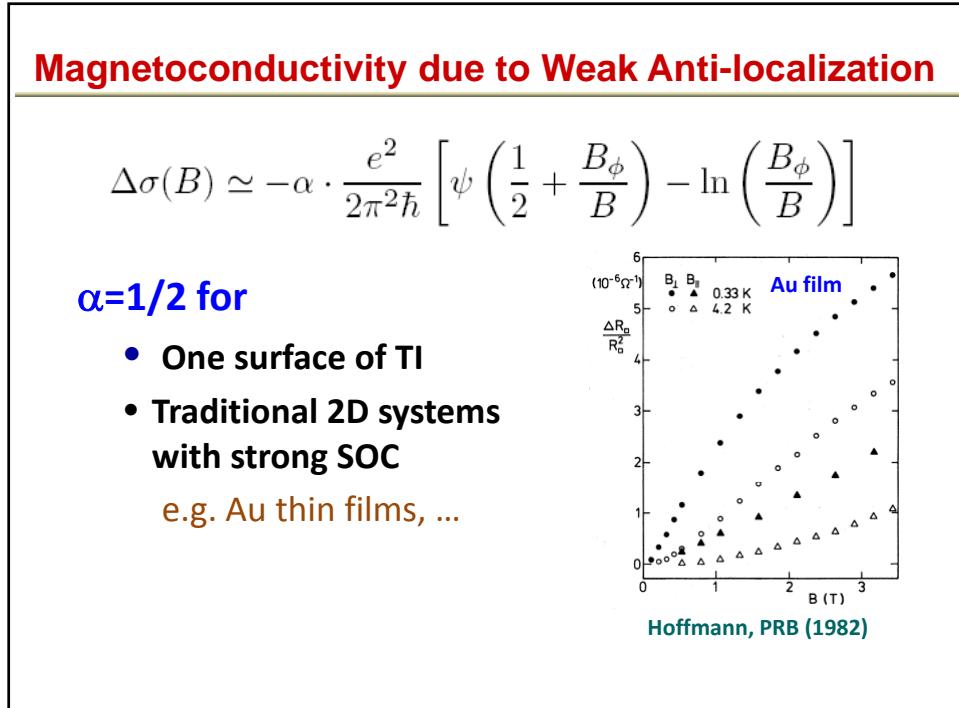


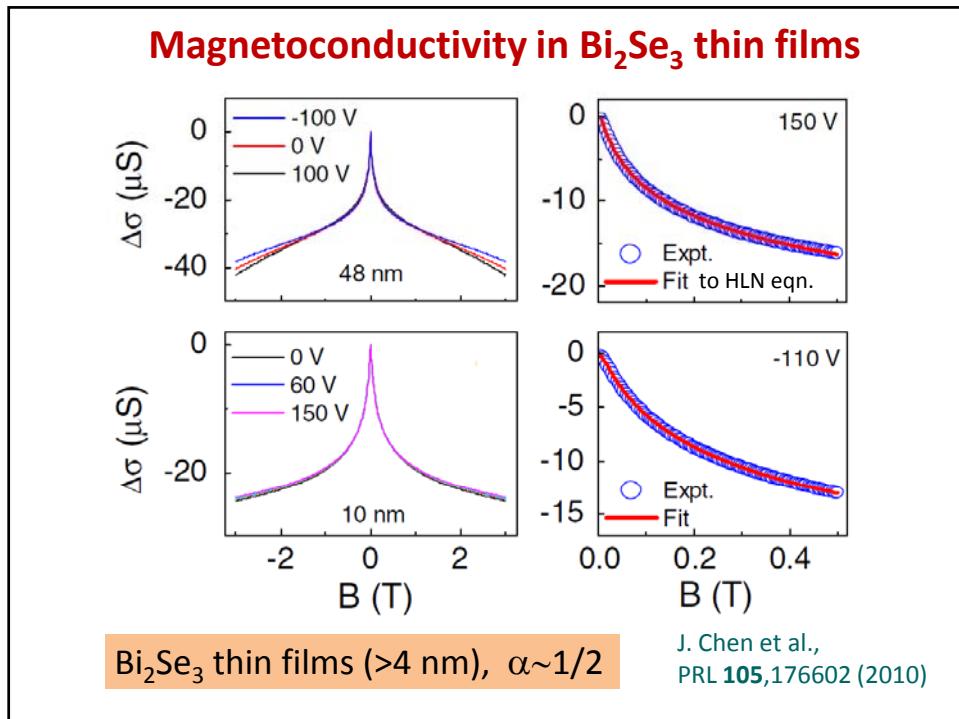
Ostrovsky,Gornyi & Mirlin, *PRL* **105**, 036803 (2010)
See also: Ostrovsky et al, *PRL* (2007); Ryu et al., *PRL* (2007);
Fu & Kane, *PRL* (2012); Konig et al., *PRB* (2012).



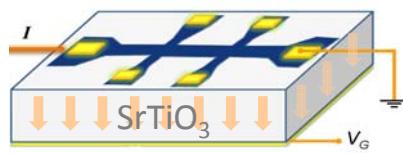


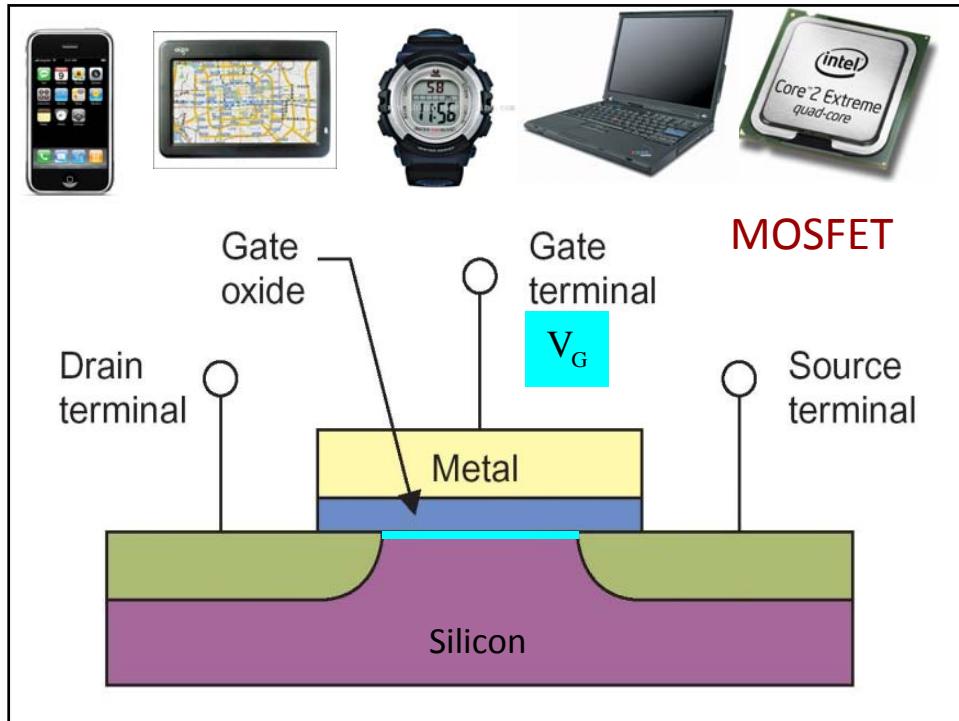
Weak anti-localization





II. Gate voltage control of chemical potential



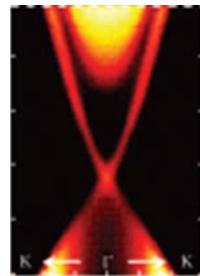


Field effect tuning of chemical potential

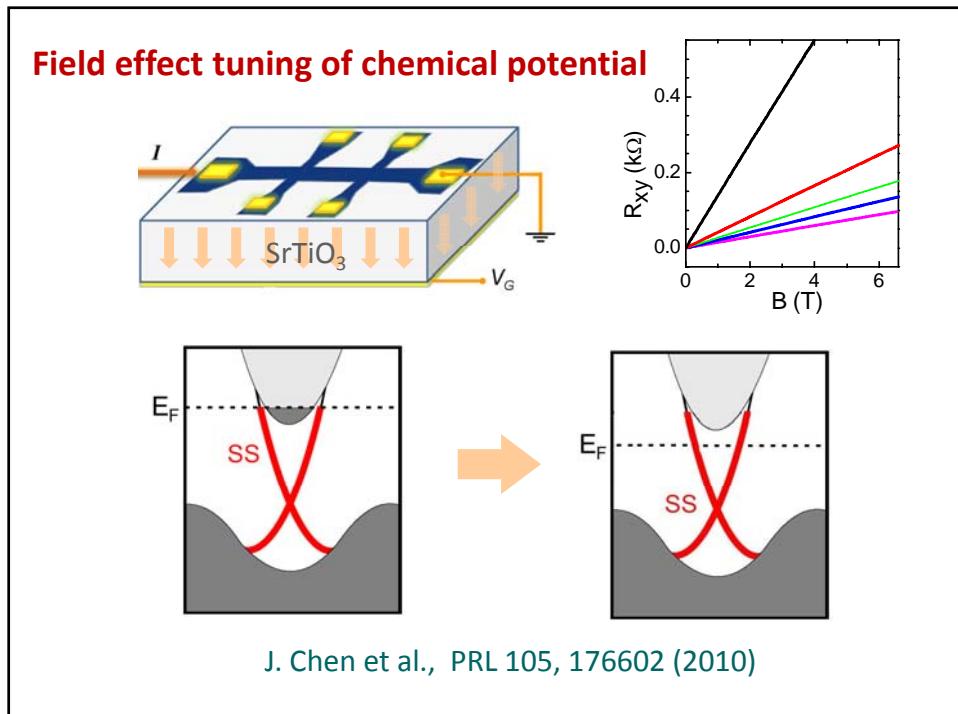
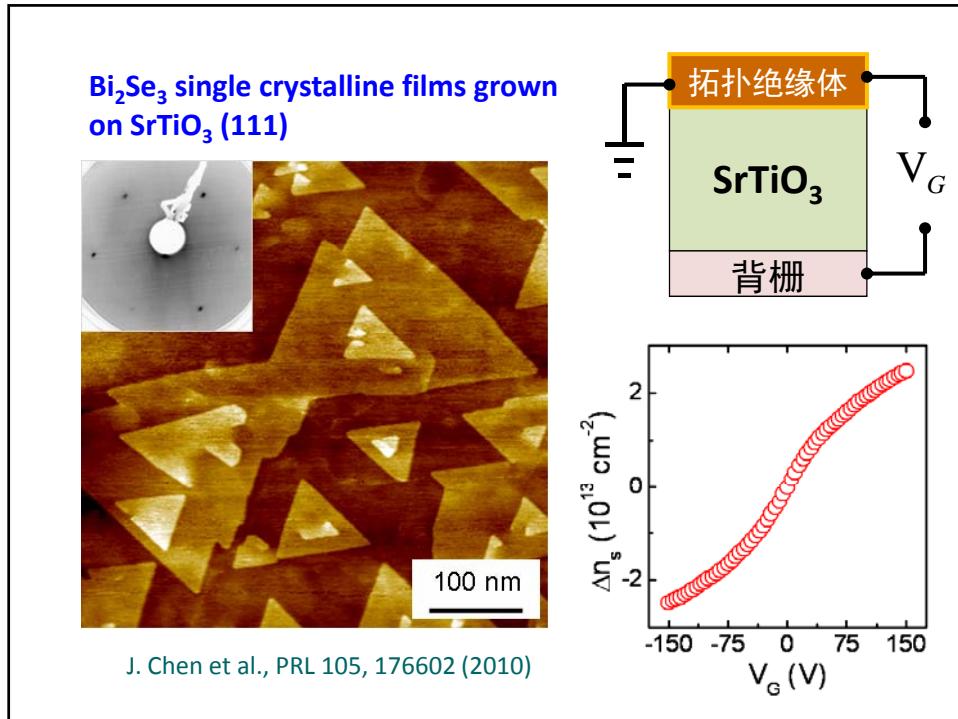
Gate voltage tuning of chemical potential

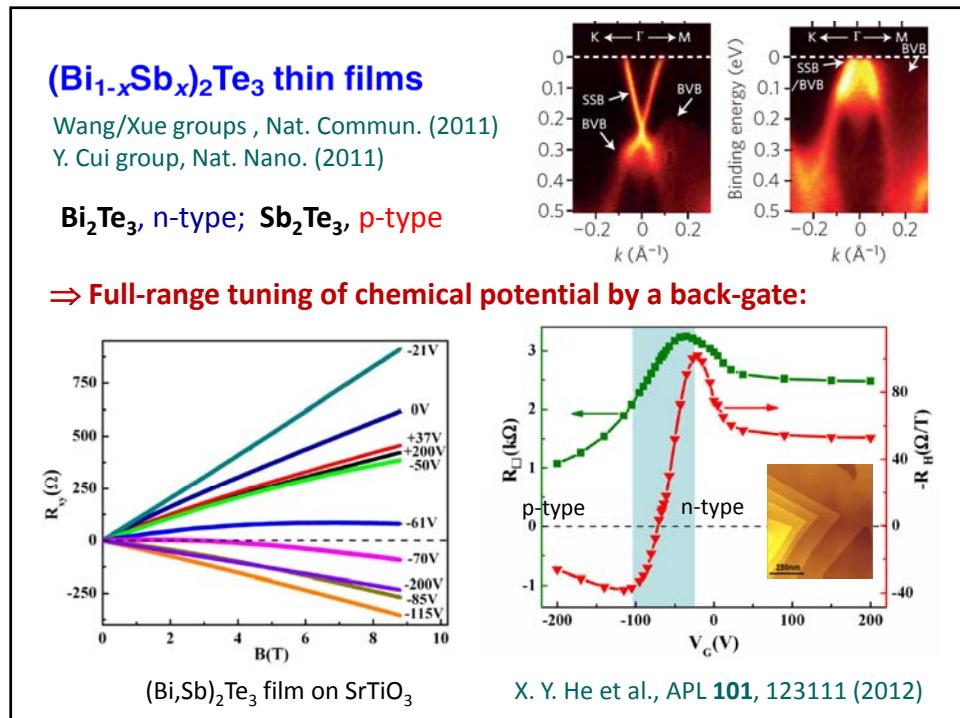
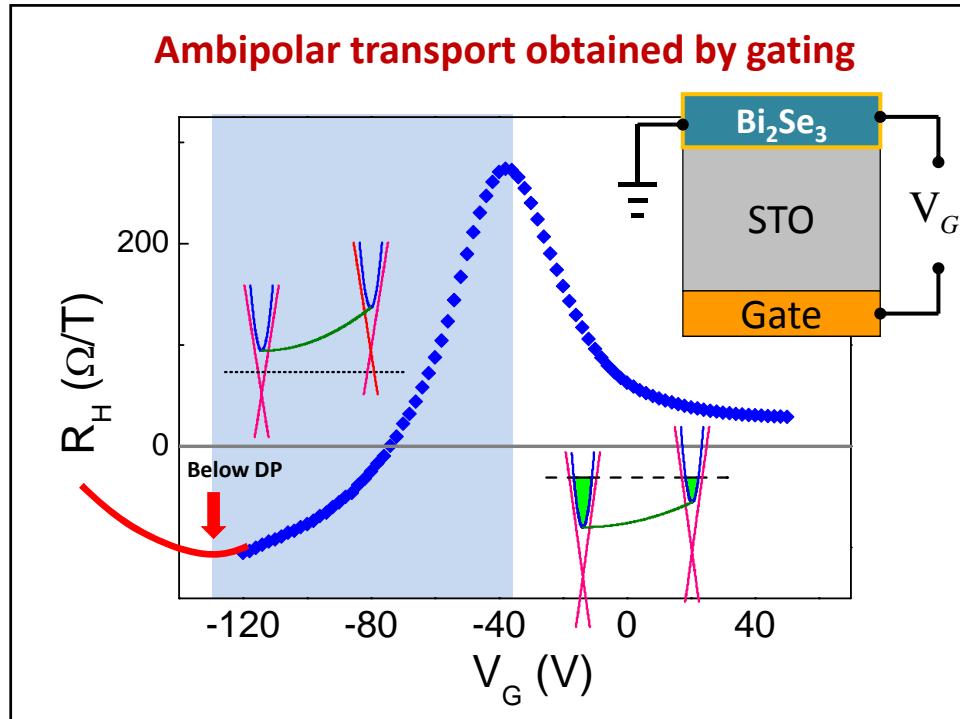
- Si, InAs, Graphene: $\sim 10^{12} \text{ cm}^{-2}$
- GaAs: $\sim 10^{11} \text{ cm}^{-2}$

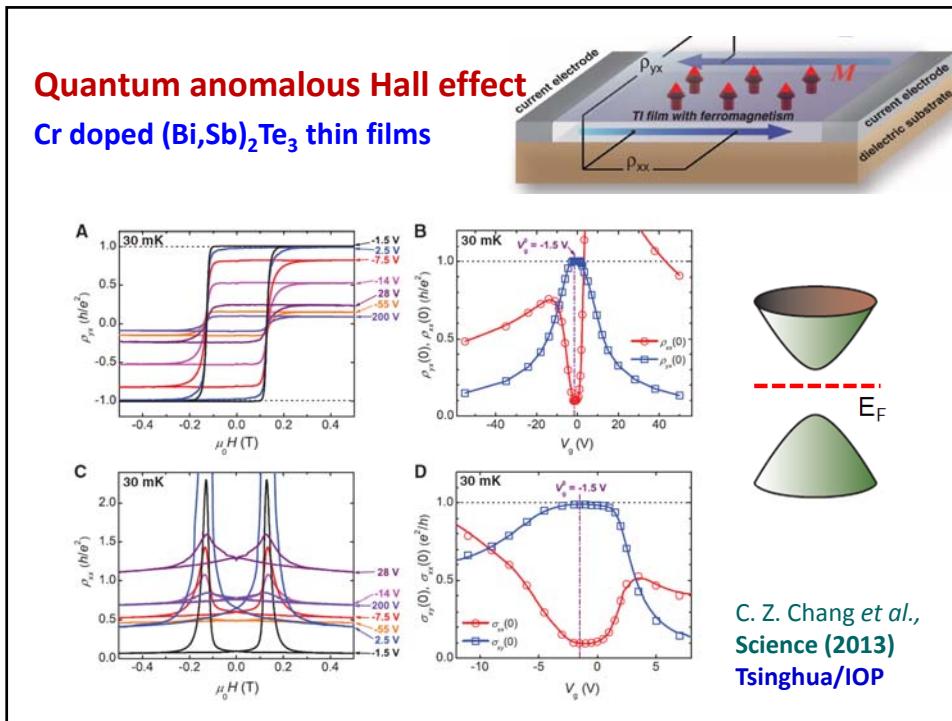
Challenge: $\Delta n_s > 2 \times 10^{13} \text{ cm}^{-2}$



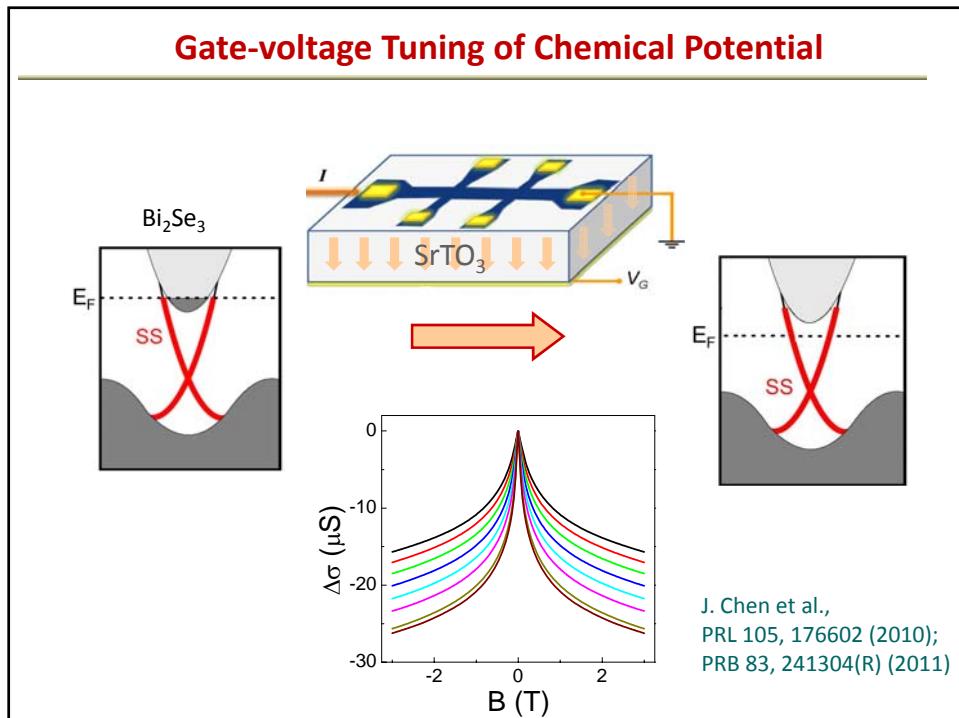
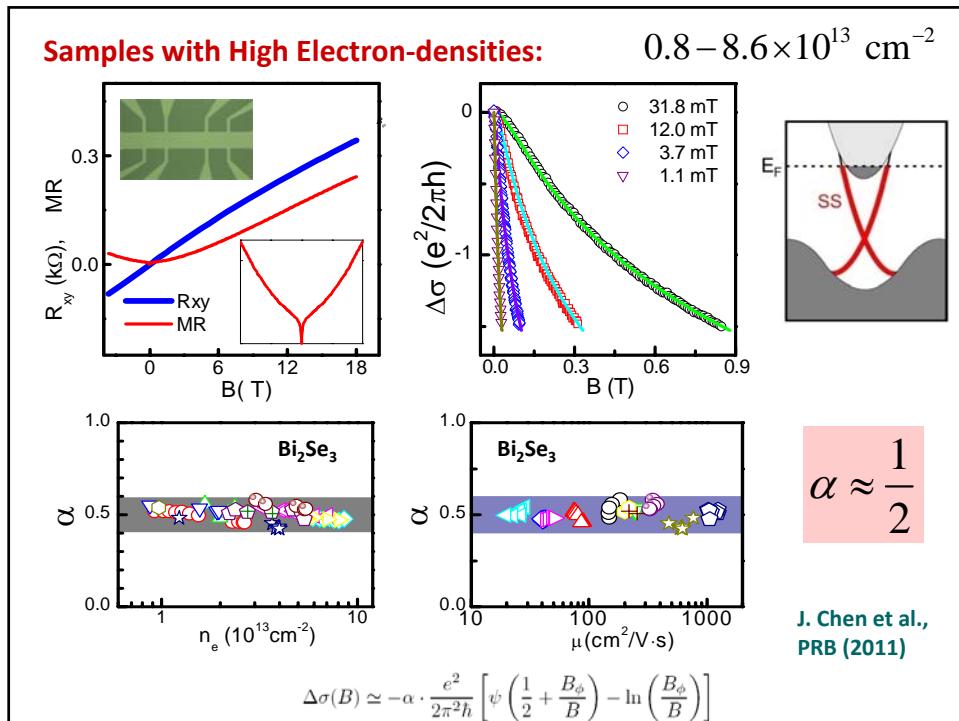
SiO_2 : $\Delta n_s < 1 \times 10^{13} \text{ cm}^{-2}$, and not suitable for epitaxial growth

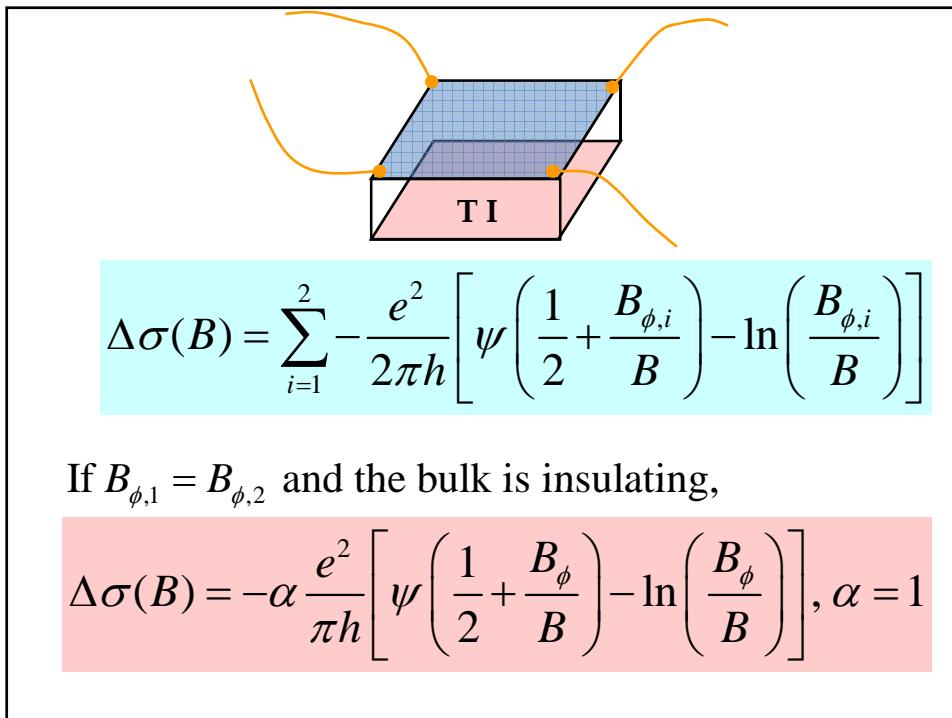
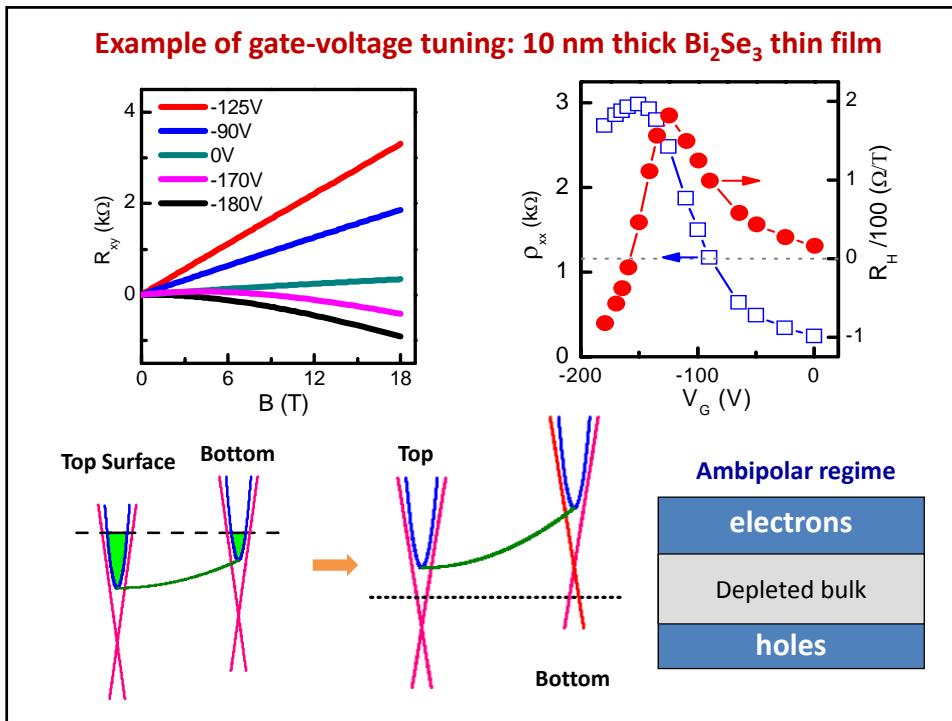




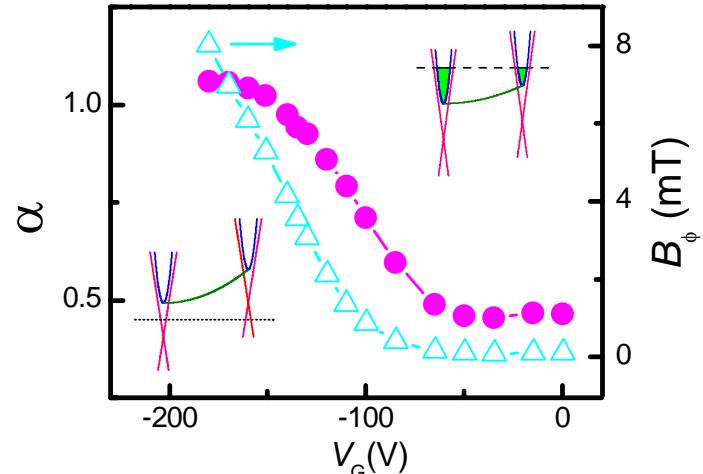


III. Probe surface transport with weak antilocalization effect





Tuning toward decoupled surface-dominating transport

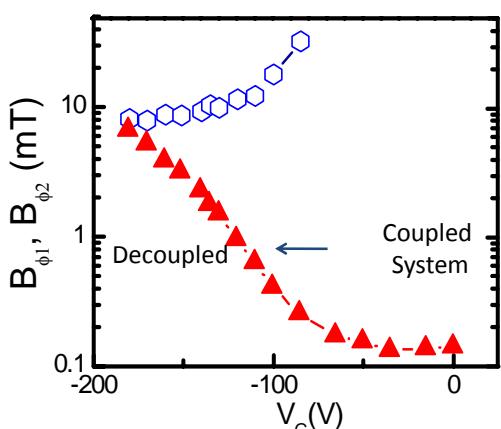


$$\Delta\sigma(B) \simeq -\alpha \cdot \frac{e^2}{2\pi^2\hbar} \left[\psi\left(\frac{1}{2} + \frac{B_\phi}{B}\right) - \ln\left(\frac{B_\phi}{B}\right) \right]$$

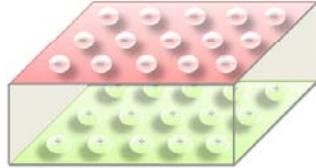
Hikami-Larkin-Nagaoka
equation (1981)

Fit with two-component HLN equation:

$$\Delta\sigma(B) = \sum_{i=1}^2 \frac{e^2}{2\pi h} \left[\psi\left(\frac{1}{2} + \frac{B_{\phi,i}}{B}\right) - \ln\left(\frac{B_{\phi,i}}{B}\right) \right]$$



Top surface: electrons



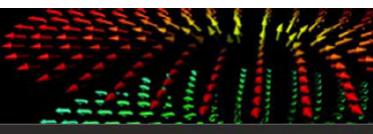
Theory e-h pair

Topological exciton condensation

Quantized vortex: $\pm e/2$ charge

Seradjeh, Moore, Franz, PRL (2009)

Journal Club for Condensed Matter Physics
A Monthly Selection of Interesting Papers by Distinguished Correspondents



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Whispers in Bedlam: Detecting the Dirac metal at a surface of a topological insulator by means of weak localization

Electrically tunable surface-to-bulk coherent coupling in topological insulator thin films.
arXiv:1104.1404 (2011)
Authors: H. Steinberg, J.-B. Laloö, V. Fatemi, J. S. Moodera, P. Jarillo-Herrero
and

Tunable surface conductivity in Bi_2Se_3 revealed in diffusive electron transport
Phys. Rev. B 83, 241304/1-4 (2011)
Authors: J. Chen, X. Y. He, K. H. Wu, Z. Q. Ji, L. Lu, J. R. Shi, J. H. Smet, and Y. Q. Li

Recommended with a Commentary by Leonid Glazman, Yale University

Garate-Glazman Theory

WAL in two coupled channels:

$$\Delta\sigma_{\perp} = -\frac{e^2}{4\pi^2\hbar} \left[f\left(\frac{B_a}{B}\right) + f\left(\frac{B_b}{B}\right) \right]$$

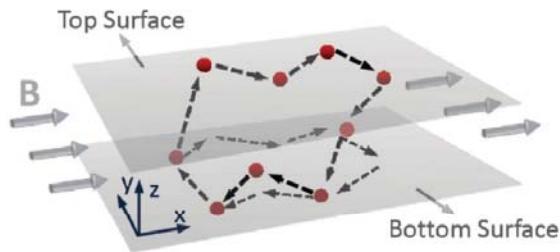
$$B_a = B_{\phi}, \quad B_b = B_{\phi} \left(1 + 2 \frac{\tau_{\phi}}{\tau_s}\right)$$

$$f(z) = \Psi(1/2 + z) - \ln(z)$$

Coherent surface-bulk scattering $\Rightarrow \alpha=1/2$

Garate & Glazman, PRB 86, 035422 (2012)

IV. Manifestation of WAL in parallel magnetic field transport

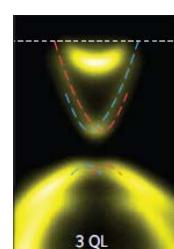
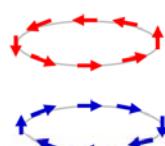
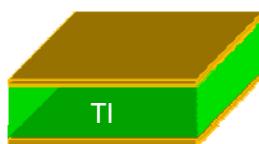


Motivation

Effort on making the bulk of the TI truly insulating: $\text{Bi}_2\text{Te}_2\text{Se}$, $(\text{Bi}, \text{Sb})_2\text{Te}_3$, $(\text{Bi}, \text{Sb})_2(\text{Se}, \text{Te})_3$, ...

resistivity up to several $\Omega \cdot \text{cm}$

Ando et al., JPSJ (2010)



Y. Zhang et al.,
Nature Phys.
(2010)

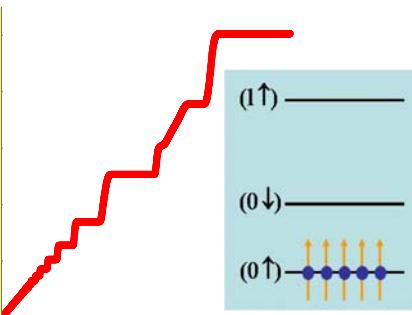
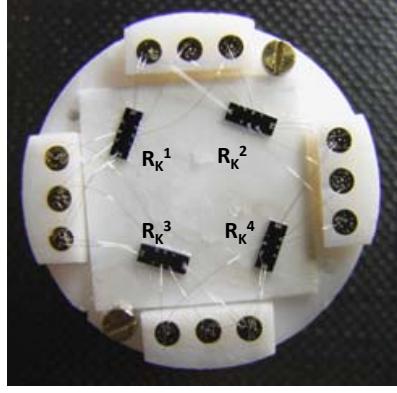
New challenges:

How to detect weak couplings between the top and bottom surfaces or between the surfaces and the bulk?

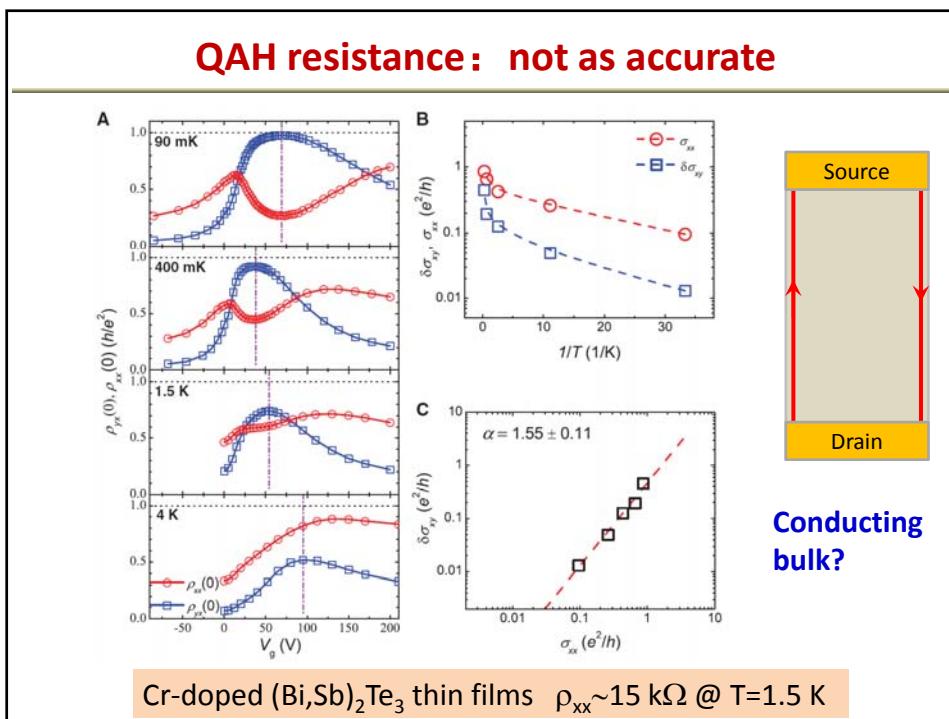
von Klitzing constant:

$$R_K = \frac{h}{e^2} = 25812.807 \Omega$$


$R_K^{1,2,3,4}$ identical within an accuracy of some parts in 10^{11}

F. Schopfer, (2006)



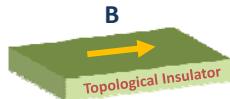
Magnetoresistance of thin films and of wires in a longitudinal magnetic field

B. L. Al'tshuler and A. G. Aronov

B. P. Konstantinov Institute of Nuclear Physics, Academy of Sciences of the USSR

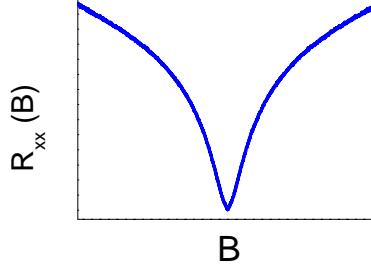
(Submitted 2 April 1981)

Pis'ma Zh. Eskp. Teor. Fiz. 33, No. 10, 515–518 (20 May 1981)



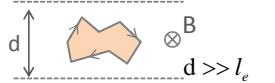
$$\Delta\sigma_{||}(B) = -\frac{e^2}{4\pi^2\hbar} \ln(1+bB^2)$$

$$b = \beta \frac{eW^2}{4\hbar B_\phi}, \quad \beta = \frac{1}{3}$$



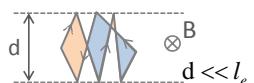
Mechanisms for magnetoresistance in parallel fields

T-dependence of β



Altshuler-Aronov (AA)
 $\beta=1/3$

weak



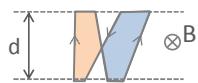
Dugaev-Khmelitskii (DK)
 $\beta=(1/16)\cdot(d/l_e) \ll 1$

weak



Beenakker-von Houten (BvH)
 $(1/16)\cdot(d/l_e) < \beta < 1/3$

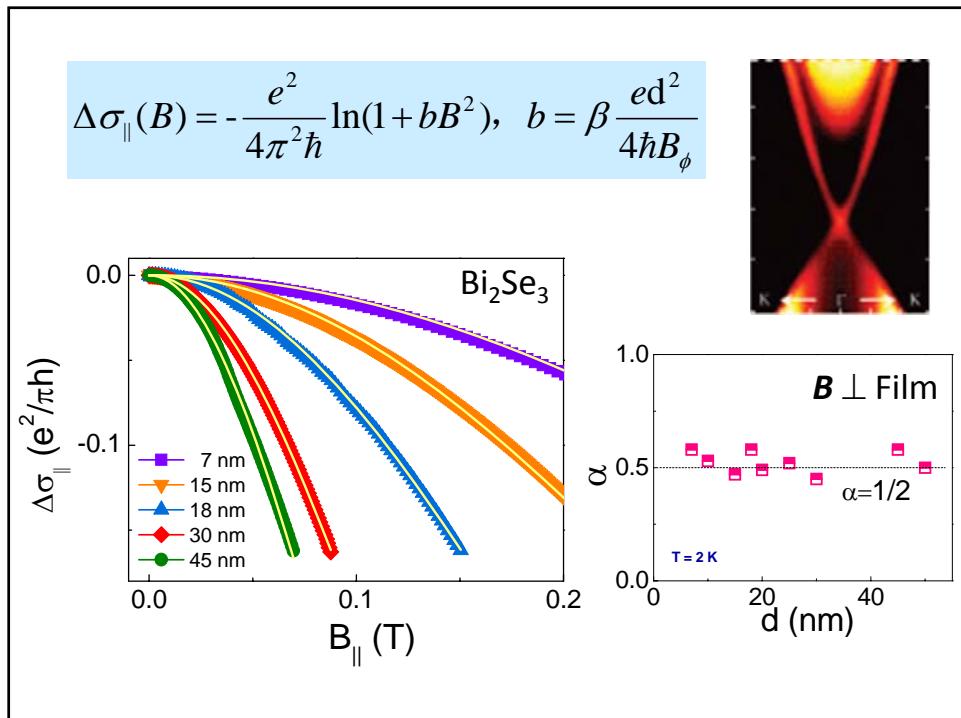
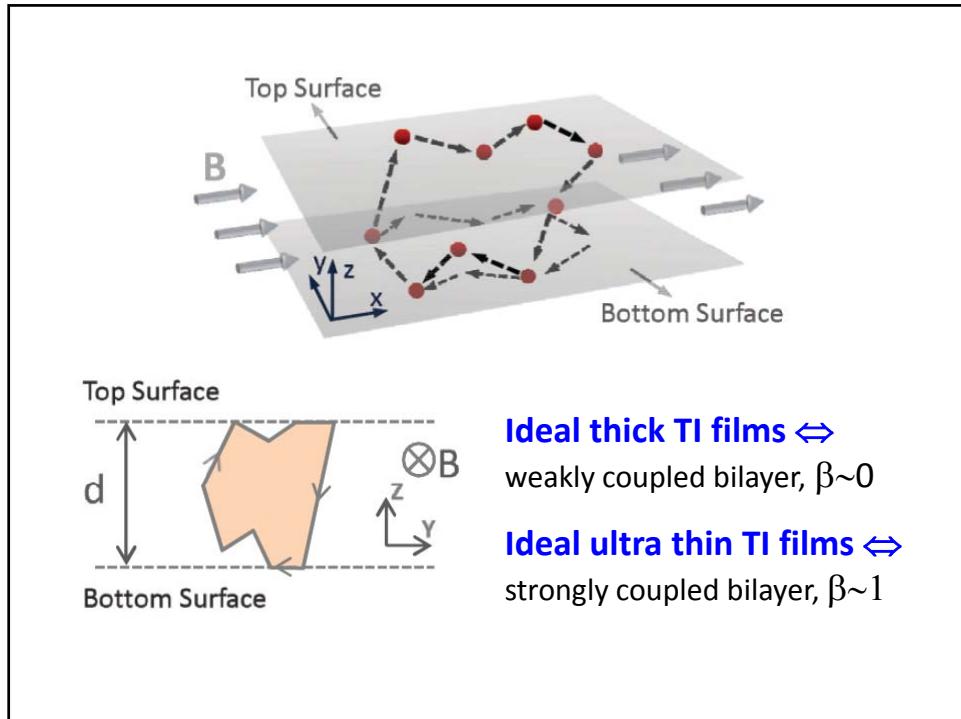
weak



Raichev-Vasilopoulos (RV)
 $0 < \beta < 1$

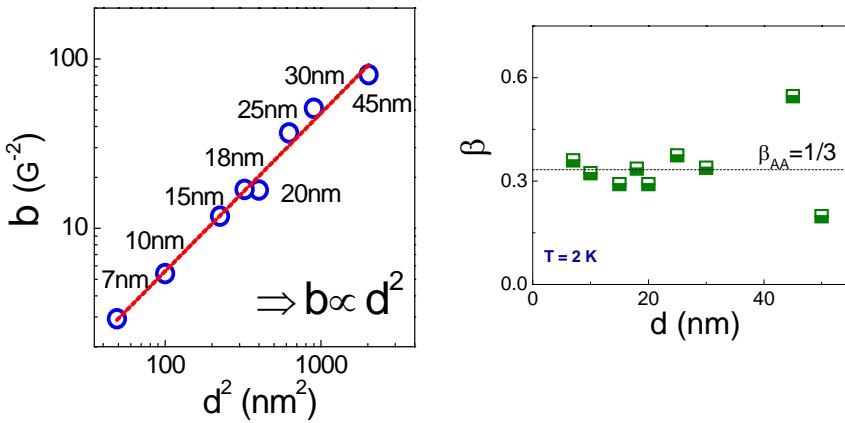
strong

$$\Delta\sigma_{||}(B) = -\frac{e^2}{4\pi^2\hbar} \ln(1+bB^2), \quad b = \beta \frac{ed^2}{4\hbar B_\phi}$$



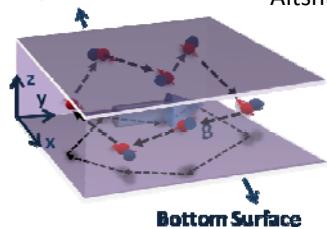
Parallel field Magnetoconductivity in Bi_2Se_3 thin films

$$\Delta\sigma_{\parallel}(B) = -\frac{e^2}{4\pi^2\hbar} \ln(1+bB^2), \quad b = \beta \frac{ed^2}{4\hbar B_\phi}$$



Topological trivial metal films

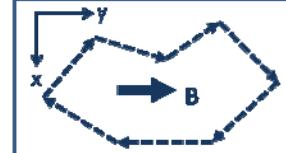
Top Surface



Altshuler-Aronov regime:

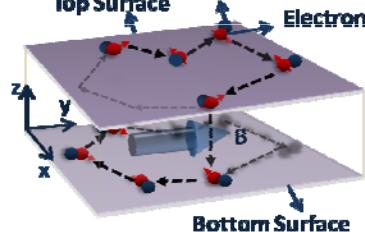
$$d \gg l_e$$

$$\beta = \frac{1}{3}$$



Topological insulator thin films

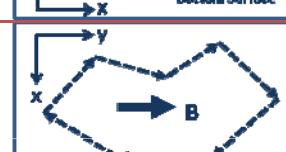
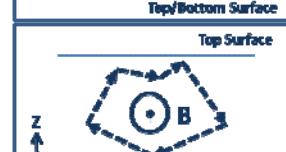
Top Surface



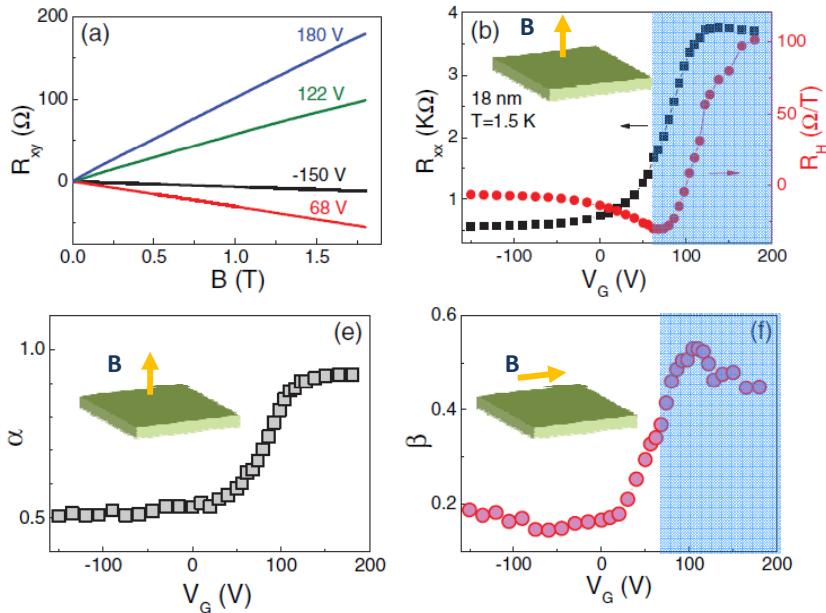
(RV regime)

$$0 \leq \beta \leq 1$$

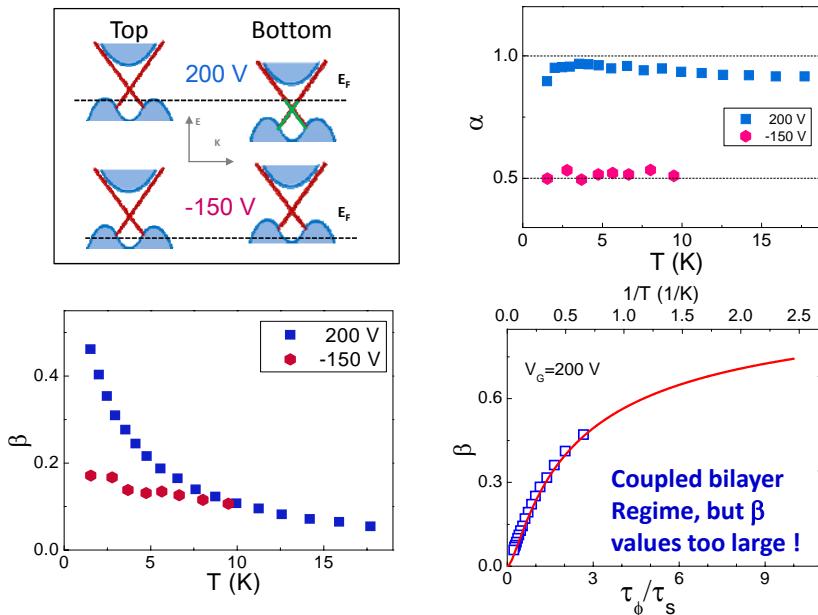
$$S = \frac{\tau_\phi}{\tau_s}$$



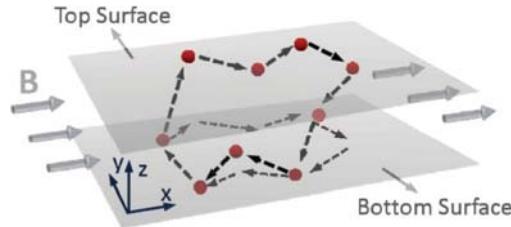
How perfect are the $(\text{Bi},\text{Te})_2\text{Te}_3$ thin films?



$(\text{Bi}_{1-x}\text{Sb}_x)_2\text{Te}_3$ thin films: T-dependence of β



Quantum diffusive transport in parallel fields



β parameter: A new figure of merit for 3D TIs?

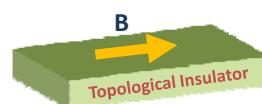
$$\beta \rightarrow 0 \text{ for ideal TIs}$$

An ultrasensitive probe for intersurface and surface-bulk couplings: $1/\tau_s \sim 10^9 \text{ s}^{-1}$

Chaojing Lin et al., PRB **88**, 041307 (R) (2013)

Summary on the WAL part

- Detecting surface state transport with perpendicular field transport
 α parameter
- Probing inter-surface couplings with parallel field transport
 β parameter

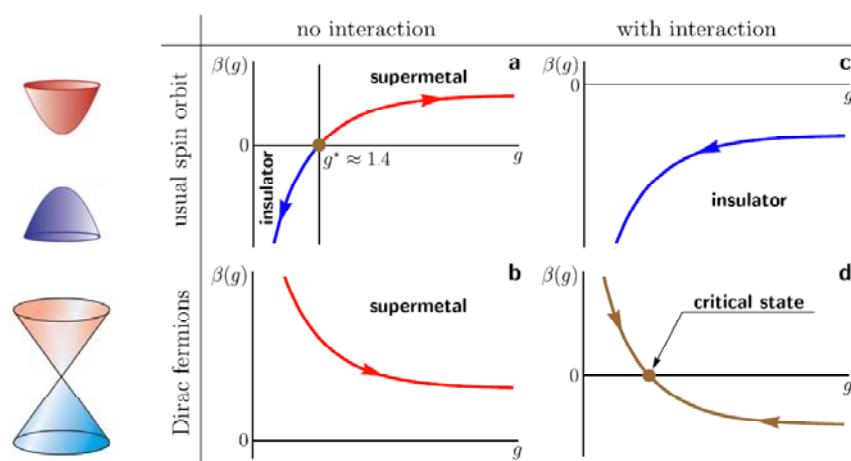


Jun Chen et al., PRB 83, 241307 (R) (2011)

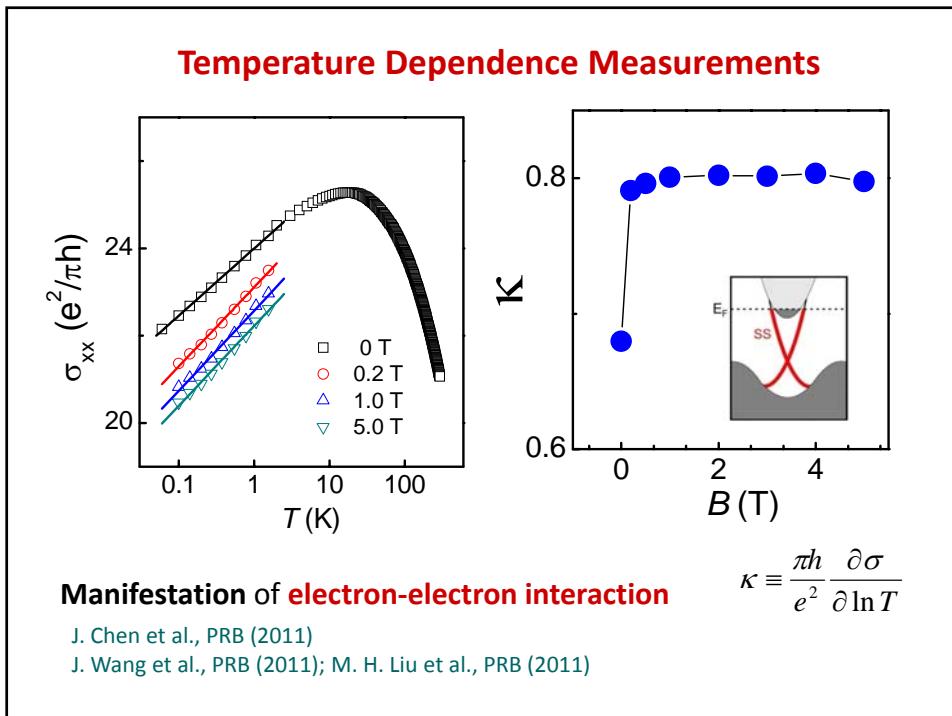
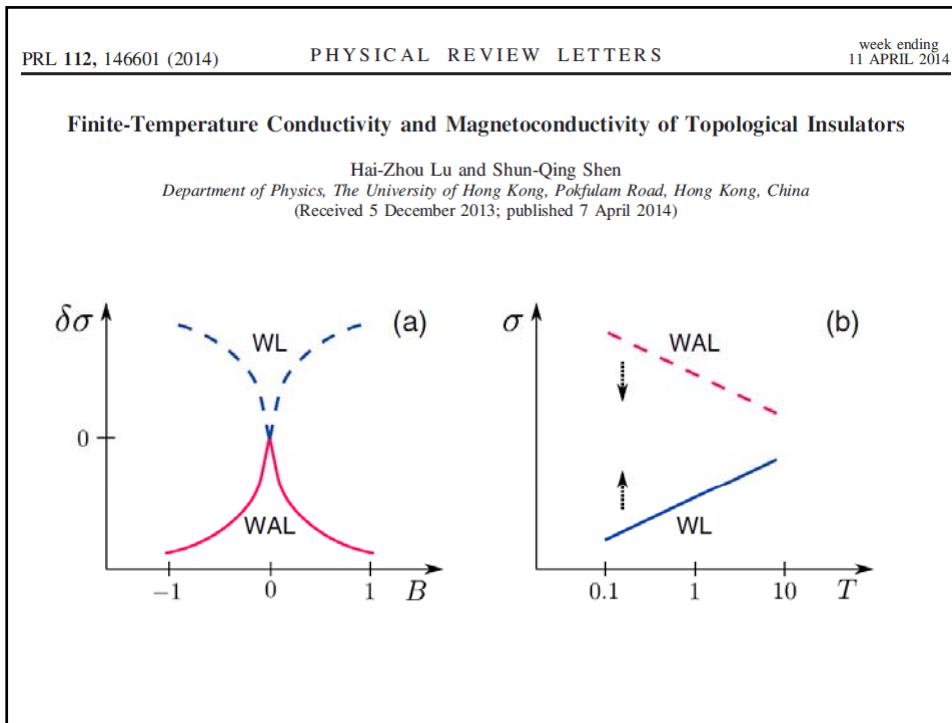
Chaojing Lin et al., PRB **88**, 041307 (R) (2013)

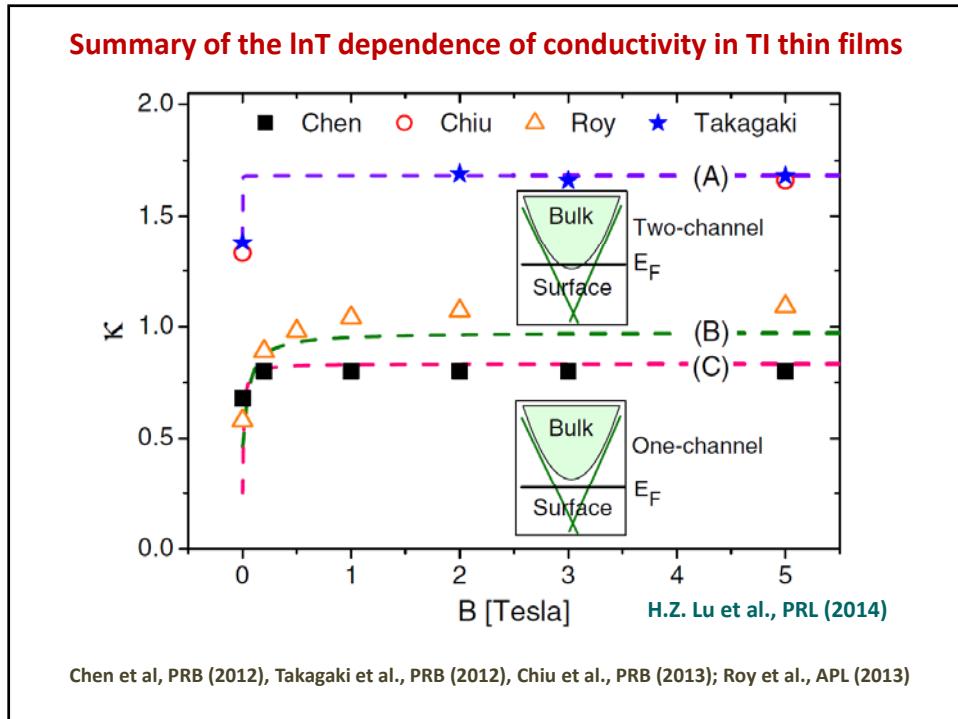
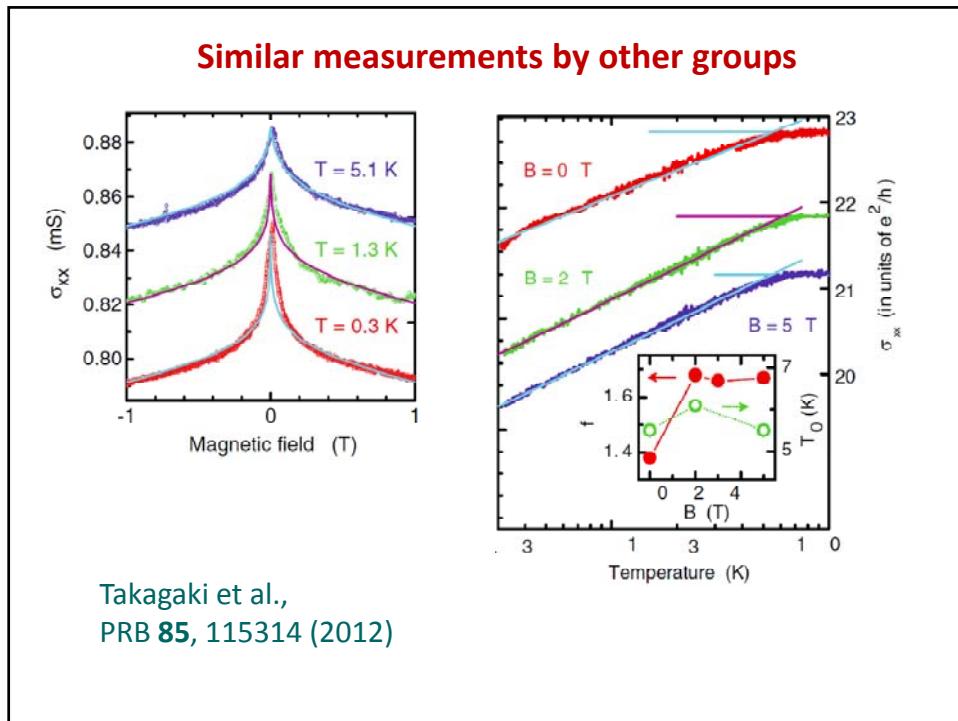
V. Temperature dependence of conductivity

- Manifestation of e-e interaction effects

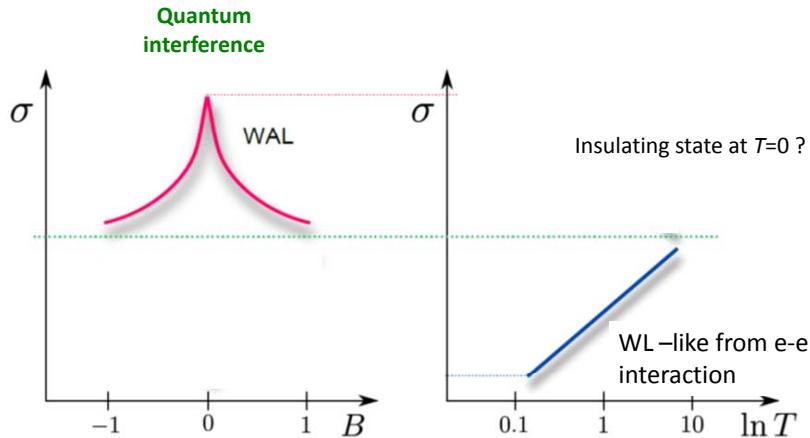


Ostrovsky, Gornyi & Mirlin, PRL 105, 036803 (2010)





Puzzling temperature dependence of conductivity



Graph from
H. Z. Lu & S. Q. Shen

Open question:
Eventual fate as $T \rightarrow 0$

PHYSICAL REVIEW B 85, 195130 (2012)

Metal-insulator transition in two-dimensional random fermion systems of chiral symmetry classes

E. J. König,^{1,2} P. M. Ostrovsky,^{3,4} I. V. Protopopov,^{3,4} and A. D. Mirlin^{1,2,3,5}

¹Institute für Theorie der kondensierten Materie, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany

²DFG Center for Functional Nanostructures, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany

³Institut für Nanotechnologie, Karlsruhe Institute of Technology, 76021 Karlsruhe, Germany

⁴L. D. Landau Institute for Theoretical Physics RAS, 119334 Moscow, Russia

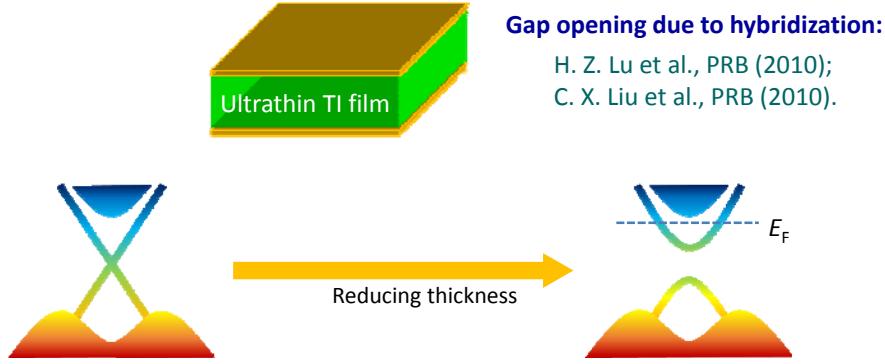
⁵Petersburg Nuclear Physics Institute, 188300 St. Petersburg, Russia

(Received 30 January 2012; published 15 May 2012)

Disordered Dirac fermions in graphene in the absence of valley mixing and **surface states of disordered topological insulators** and superconductors are characterized by sigma models with Wess-Zumino or θ terms, ensuring a **topological protection from localization**.

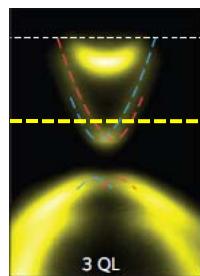
Ongoing experimental efforts, but challenges from residue bulk conductivity, weak e-e interactions, ...

VI. Taking a Different Route: Ultrathin TI films



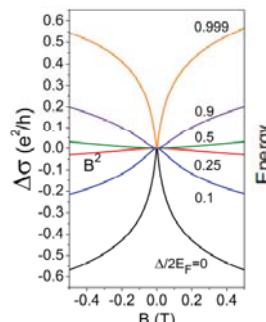
1. Transition from 3D TI to 2D TI or topologically trivial insulator
2. Anderson localization of surface states?
3. Crossover between **weak antilocalization** and **weak localization**

Predicted crossover between WAL and WL



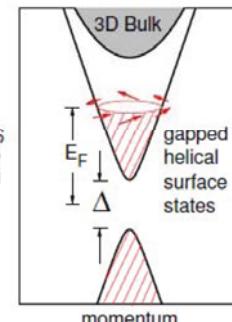
Evidence for a surface state hybridization gap from ARPES

Y. Zhang et al.,
Nature Phys. (2010)



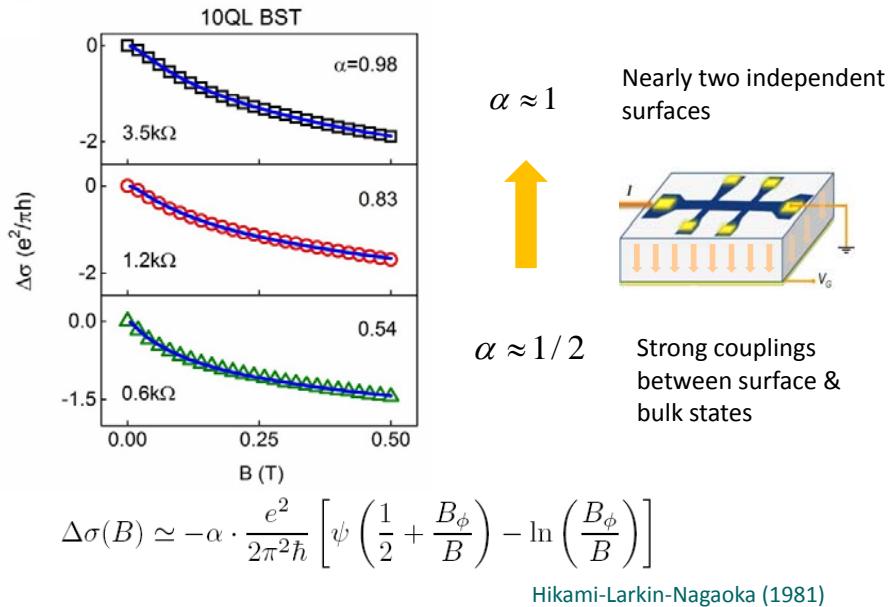
$$\text{Modified Berry's phase: } \pi \left(1 - \frac{\Delta}{2E_F} \right)$$

H.-Z. Lu, J. R. Shi & S.-Q. Shen, PRL (2011)

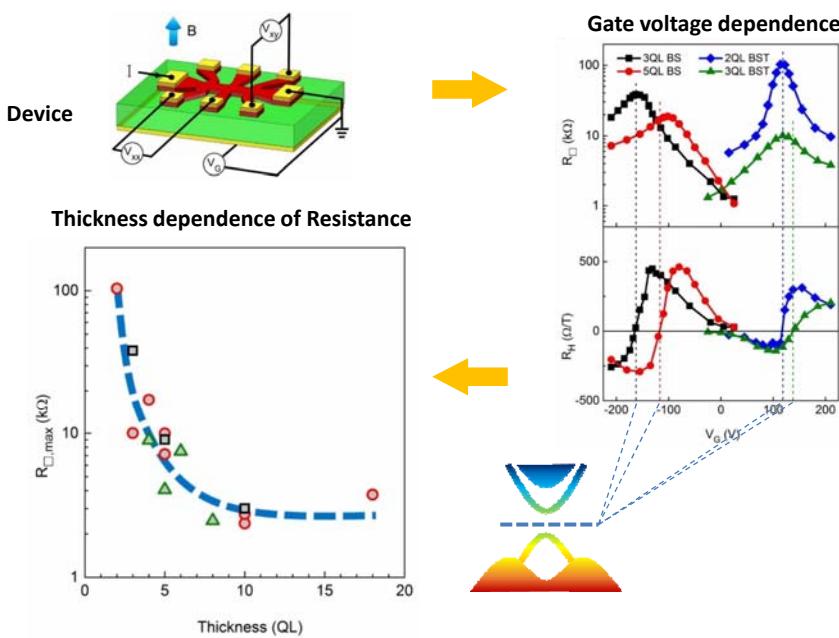


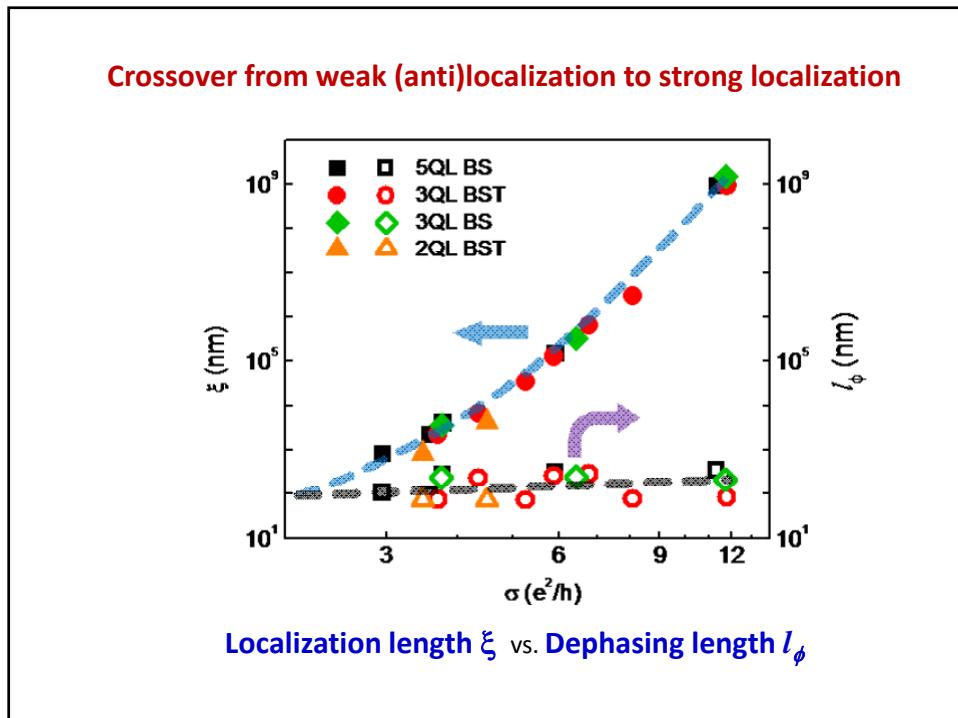
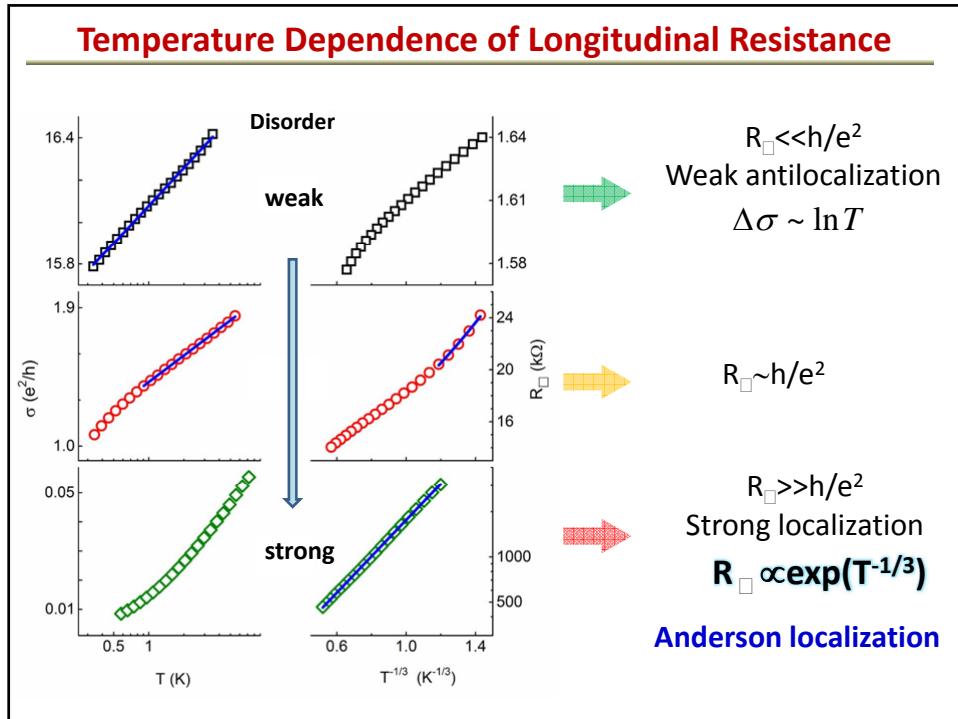
But only valid at the diffusive regime, i.e. $k_F l \ll 1$

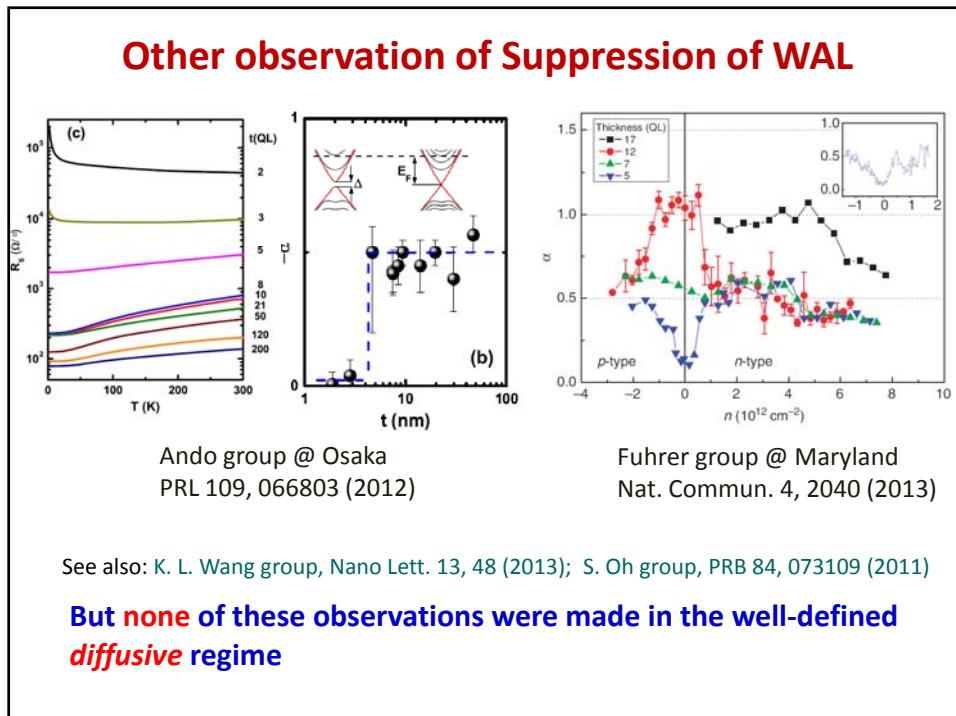
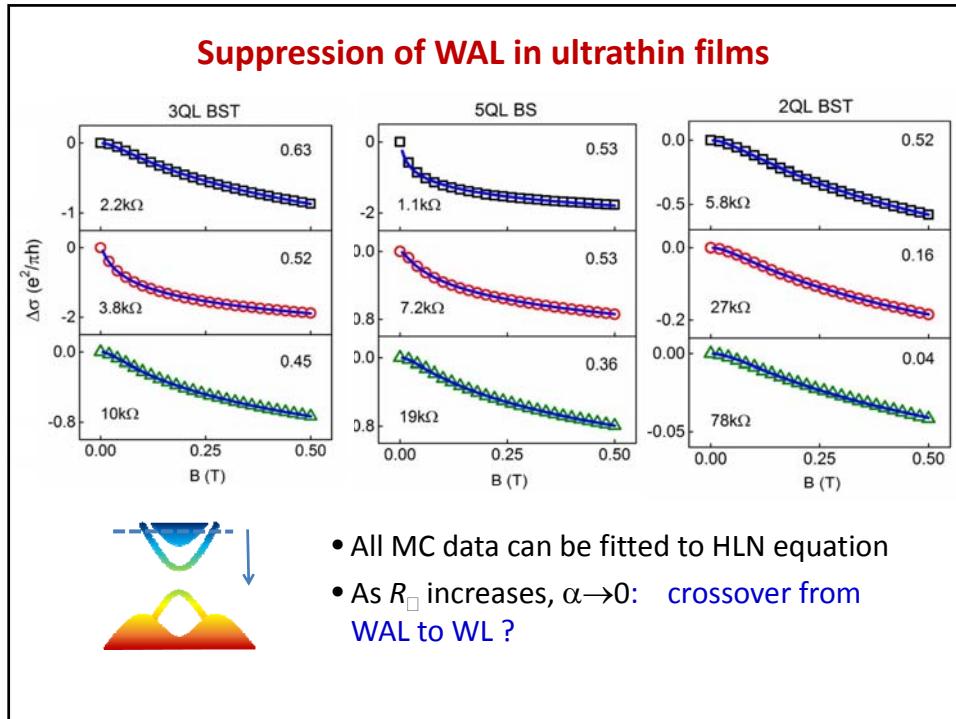
More reliable tuning of the phase coherent transport

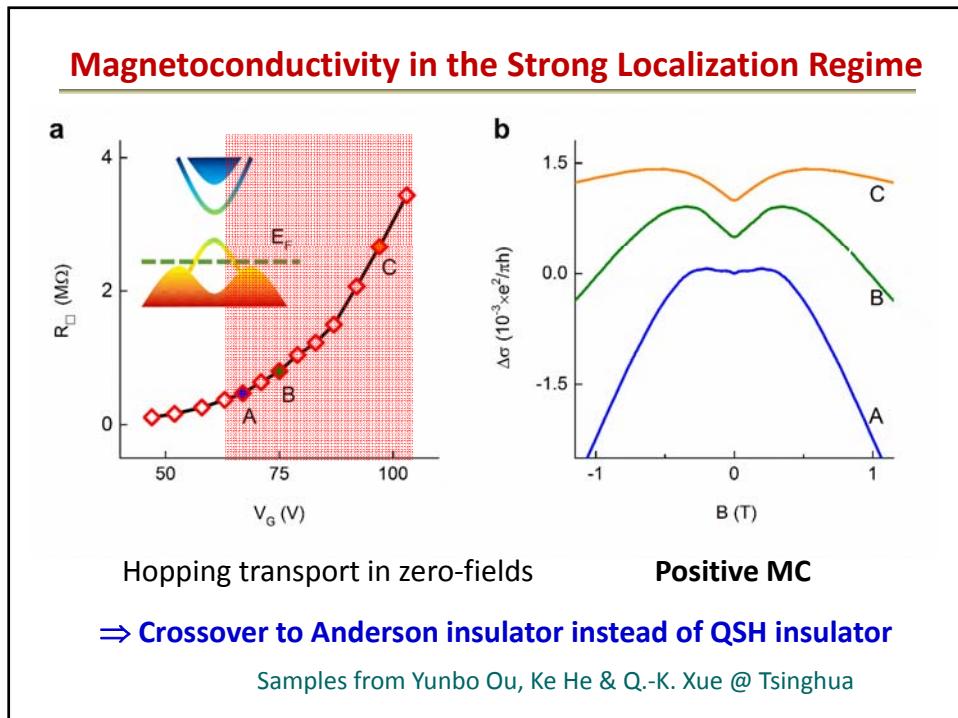
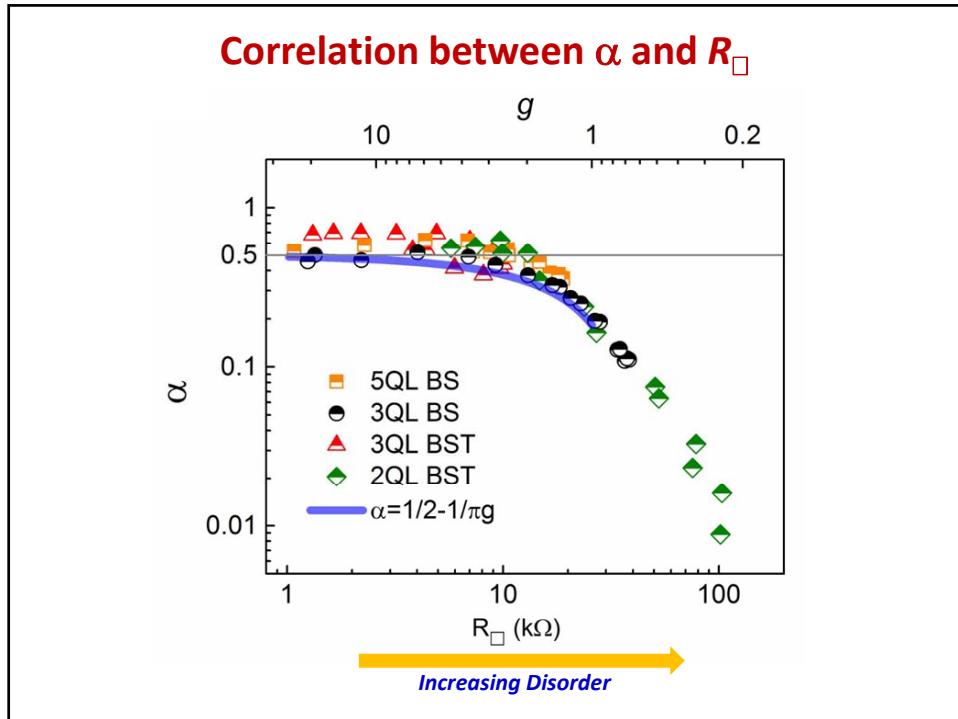


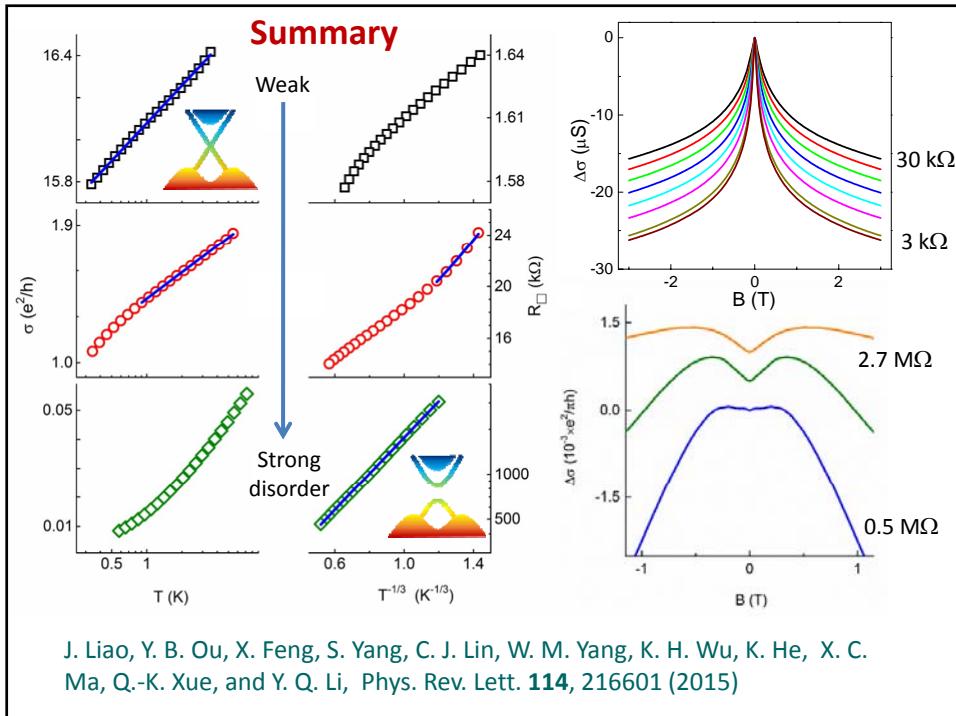
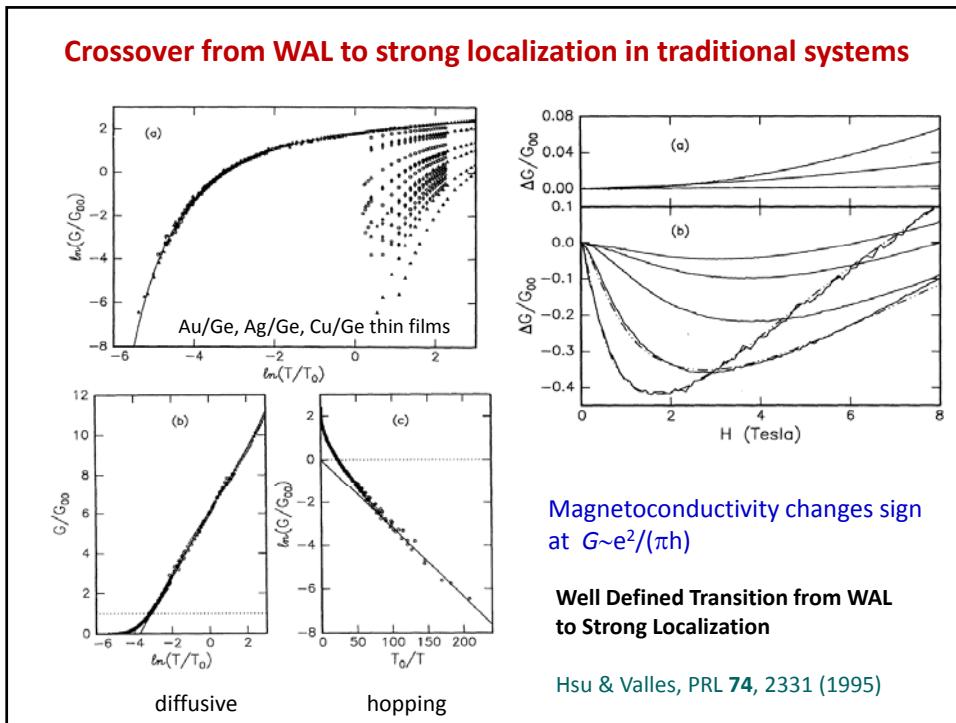
(Bi,Sb)₂Te₃ and Bi₂Se₃ Thin Films with Highly Tunable Chemical Potential











Summary & outlook

- **Truly 3D TI thin films:**
 - Only WAL-like magnetoconductivity has been observed
 - Low T conductivity only shows weakly insulating behavior due to overwhelming e-e interactions
- **Ultrathin 3D TI films:**
 - Transport in both WAL and SL regimes
 - No evidence available for transition to QSH phase

Many open questions still exist regarding the electron transport in topological insulators.

Part Two:

Evidence for half-metallicity in n- HgCr_2Se_4

Collaborators:

Tong Guan, Chaojing Lin, Chongli Yang, **Youguo Shi**, Cong Ren,
Hongming Weng, Xi Dai, Zhong Fang, IOP-CAS

Peng Xiong, Florida State University

Shishen Yan, Shandong University

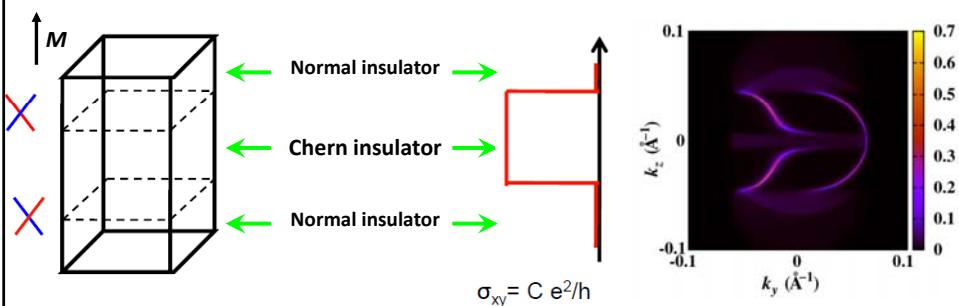
T. Guan et al., arXiv: 1503.03190, accepted by Phys. Rev. Lett.

Topological States of Matter

	2D	3D
Time-reversal symmetry	Quantum Spin Hall Insulator HgTe/CdTe QW	Topological Band/Crystalline/Kondo /Anderson/Mott Insulator
TR-symmetry broken	Quantum Hall states; Quantum Anomalous Hall States Cr-doped $(\text{Bi},\text{Sb})_2\text{Te}_3$?

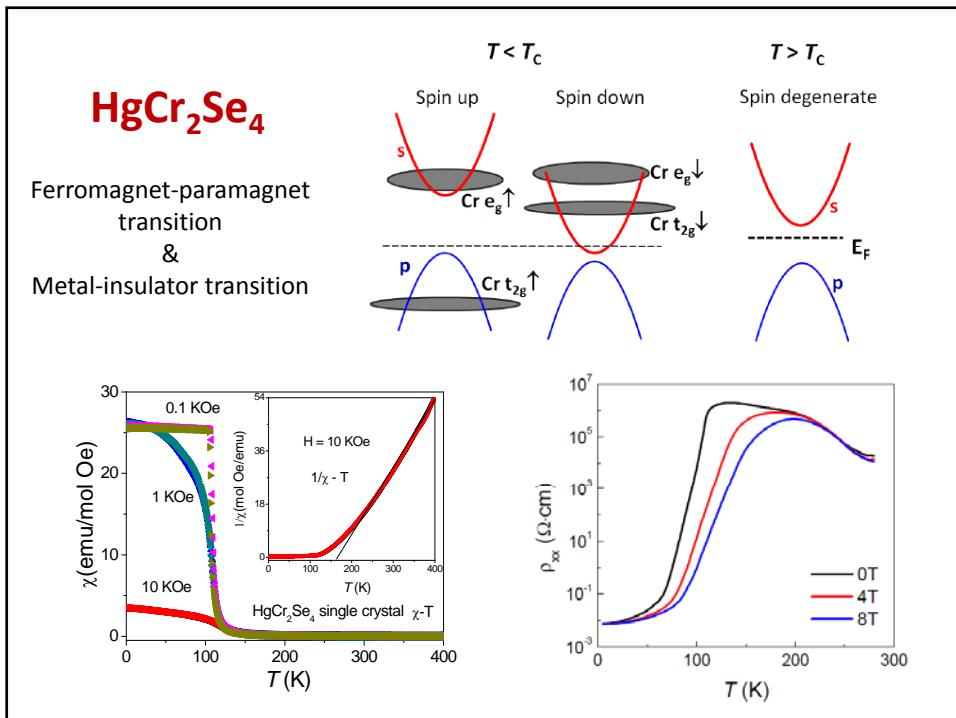
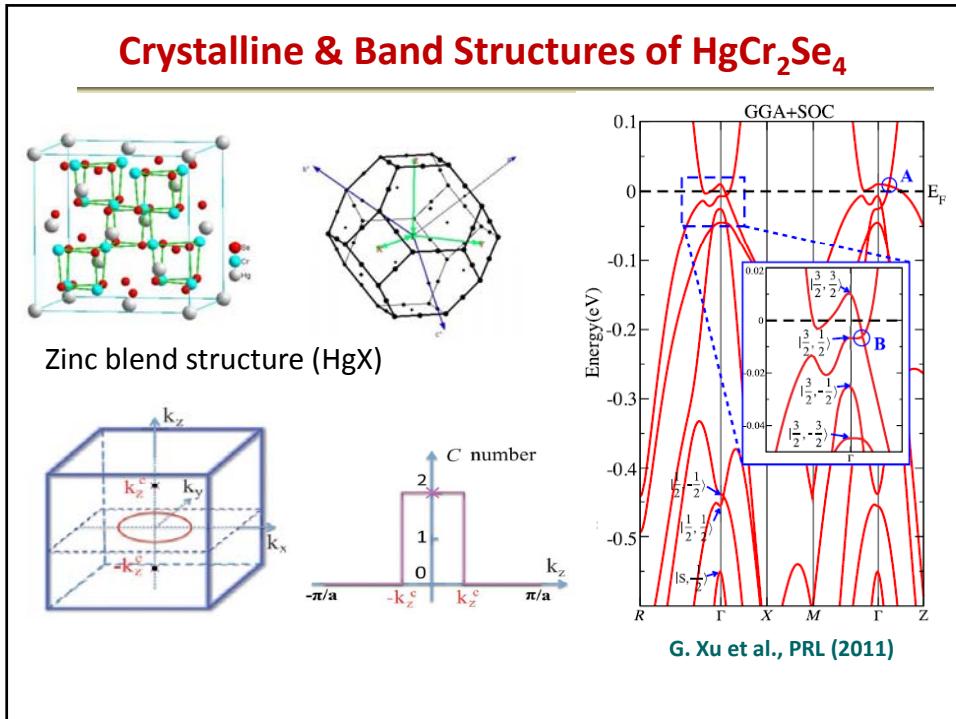
Xiangang Wan et al., PRB (2010) $\text{A}_2\text{Ir}_2\text{O}_7$

A Candidate for Magnetic Weyl Semimetals



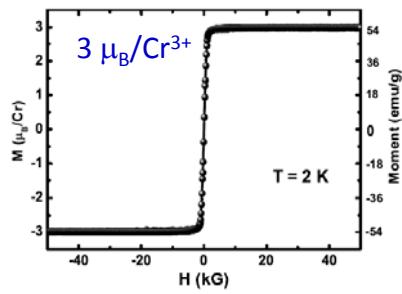
- **Fermi arcs** on surfaces
- **Quantum anomalous Hall effect** from the quantum well structure

G. Xu et al., PRL 107, 186806 (2011)

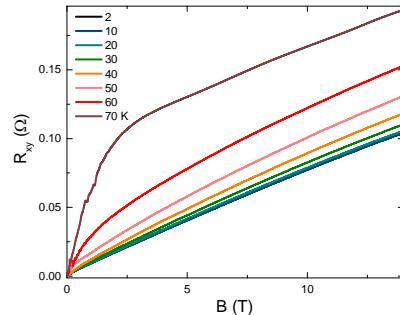


Ferromagnetic ground states of $n\text{-HgCr}_2\text{Se}_4$

Magnetism



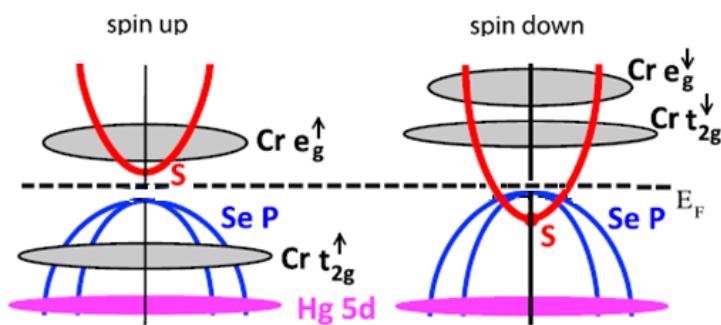
Transport



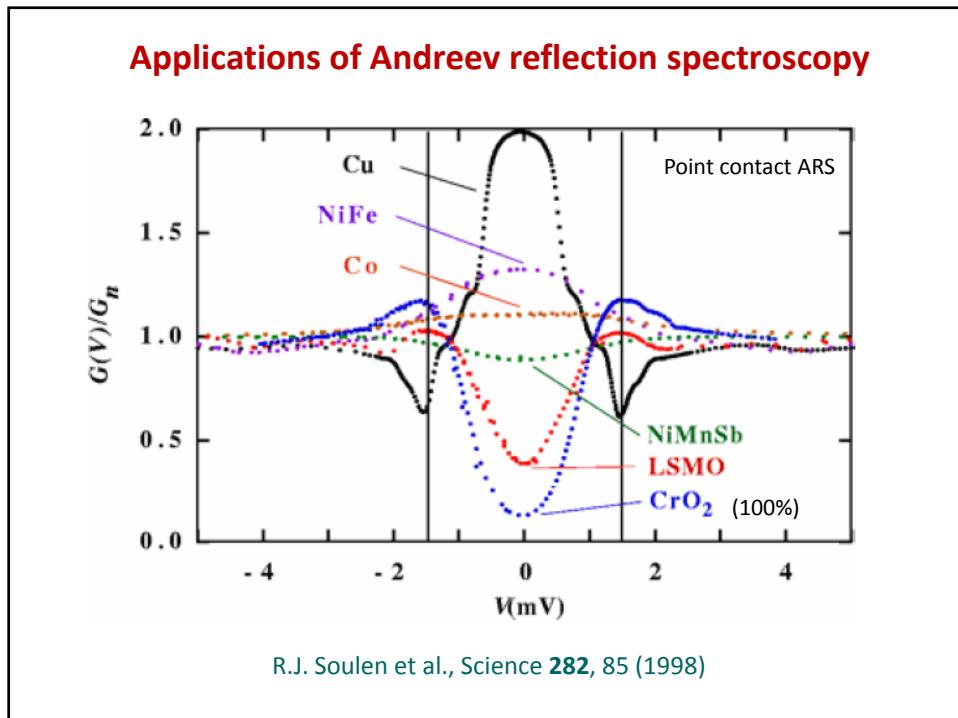
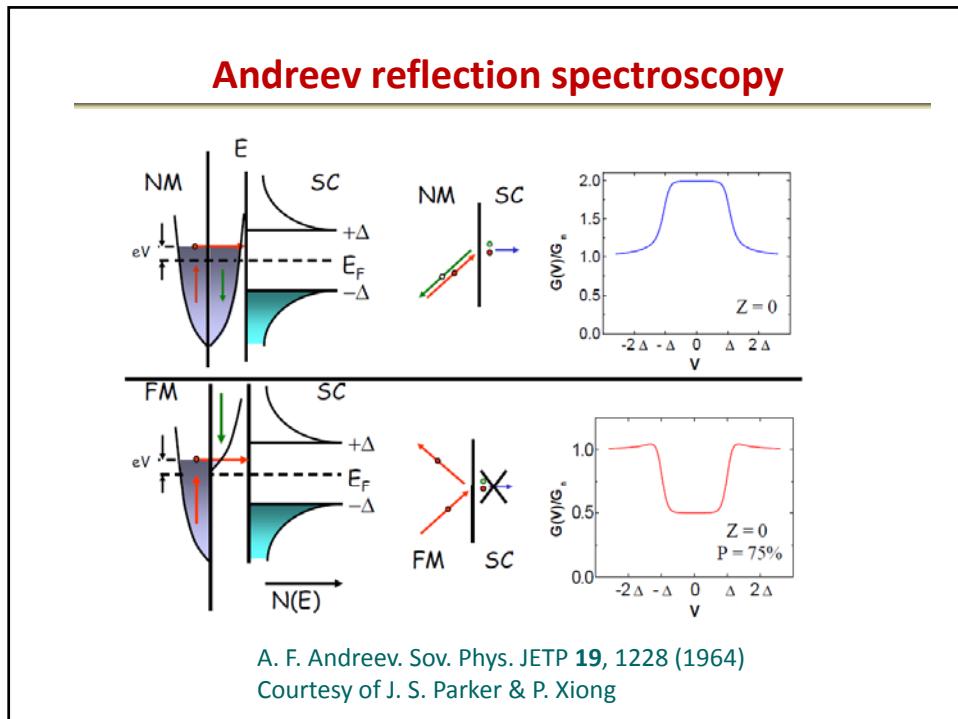
Integer Bohr magneton/unit cell:
Indication for half-metallicity

Anomalous Hall effect

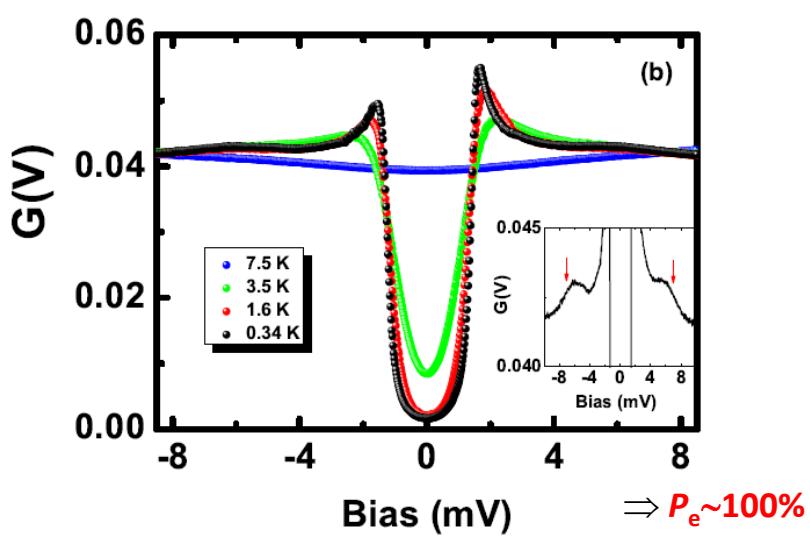
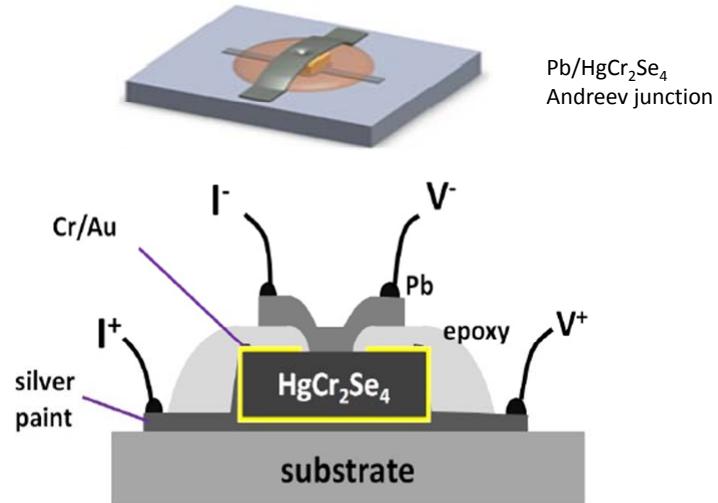
Spin polarization in $n\text{-HgCr}_2\text{Se}_4$: 100%?



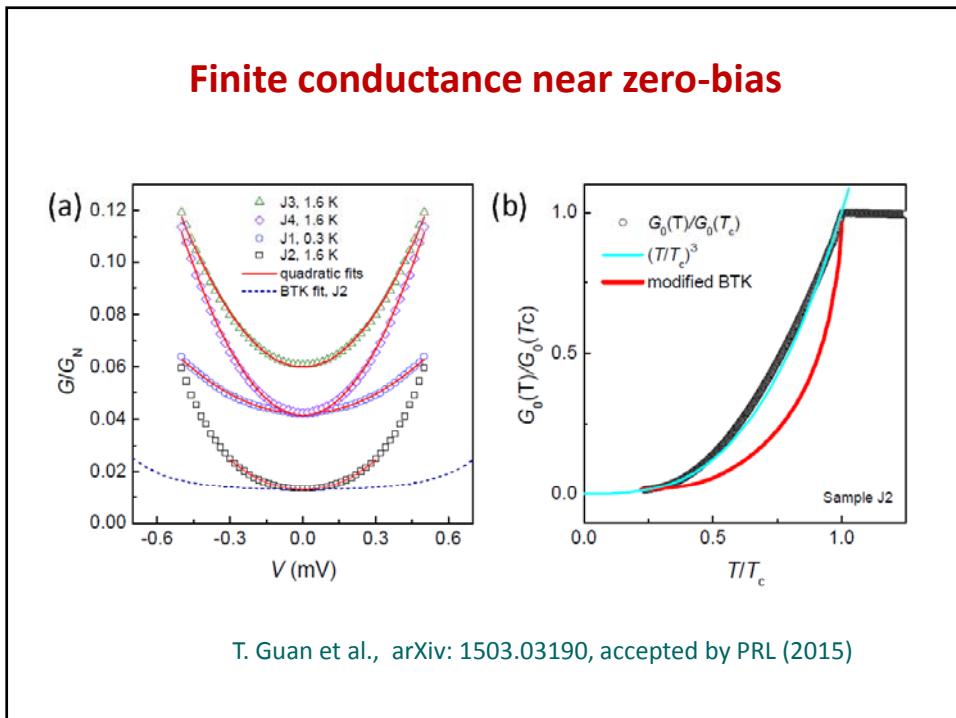
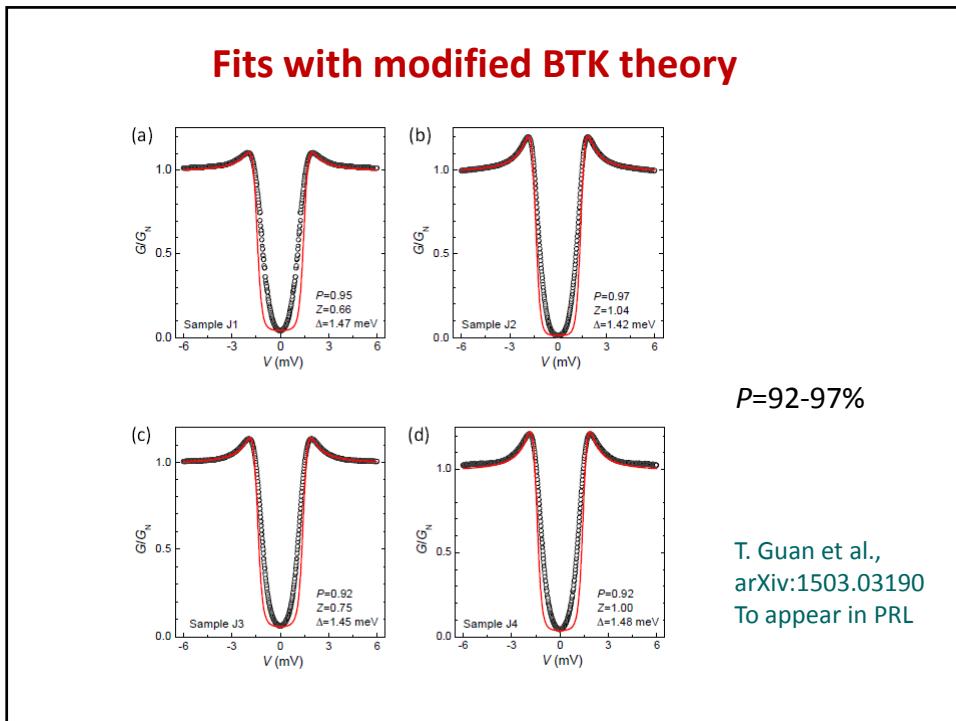
$T < T_C$, half-metallic ferromagnet
 $T > T_C$, paramagnet



Planar junctions for Andreev reflection



n-HgCr₂Se₄: low electron density, half-metallic magnetic semiconductor



Is CrO_2 Fully Spin Polarized? Analysis of Andreev Spectra and Excess Current

Tomas Löfwander,¹ Roland Grein,² and Matthias Eschrig^{2,3}

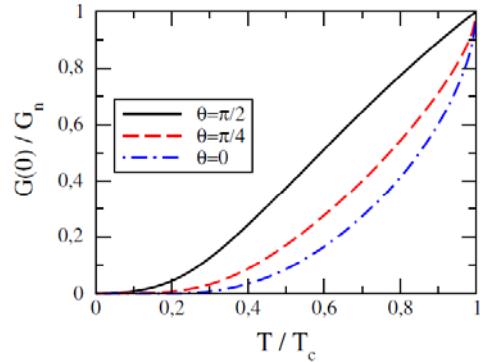
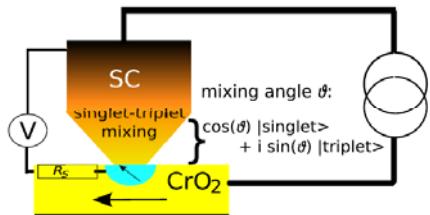
¹Department of Microtechnology and Nanoscience—MC2, Chalmers University of Technology, SE-412 96 Göteborg, Sweden

²Institut für Theoretische Festkörperphysik and DFG-Center for Functional Nanostructures, Karlsruhe Institute of Technology, D-76128 Karlsruhe, Germany

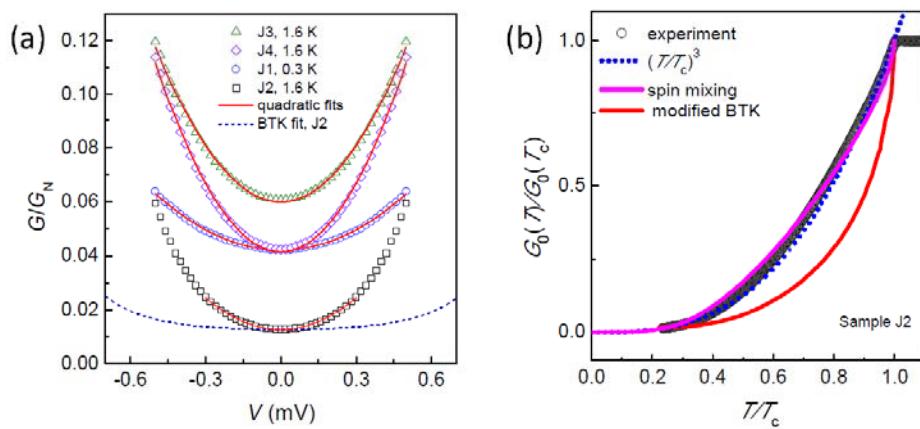
³Fachbereich Physik, Universität Konstanz, D-78457 Konstanz, Germany

(Received 19 July 2010; published 10 November 2010)

Spin active scattering model

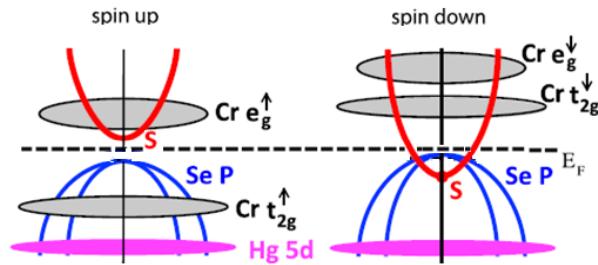


Differential conductance of $n\text{-HgCr}_2\text{Se}_4$



T. Guan et al., arXiv: 1503.03190, accepted by PRL (2015)

n-HgCr₂Se₄: s-band half-metal



Other half-metallic materials:

CrO₂, NiMnSb, (La,Sr)MnO₃: 3d electrons

EuO, EuS, ... 5d electrons

(Ga,Mn)As, only *p*-type

Outlook: Spintronics, unconventional superconductivity,
high-*T* quantum anomalous Hall effect