# Dephasing and Disorder Effects in Topological Systems

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# Outline

- ➢ Introduction
  - ❑ From quantum Hall systems to topological insulators
- ➢ Dephasing effects in topological insulators
  - ❑ Edge states of 2D quantum spin Hall insulators
  - ❑ Surface states of 3D topological insulators
- ➢ Disorder effects in topological insulators
  - ❑ Tunable Anderson transition in 2D quantum spin Hall insulators
  - ❑ Multiple Anderson transition in Weyl semimetals

# Topology

❑ Topology： an area of mathematics concerned with the properties of space that are preserved under continuous deformations.

Topological equivalent

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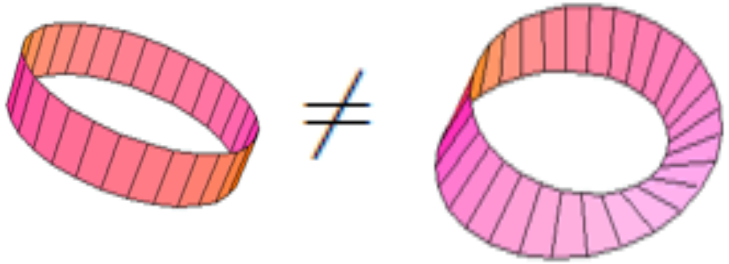
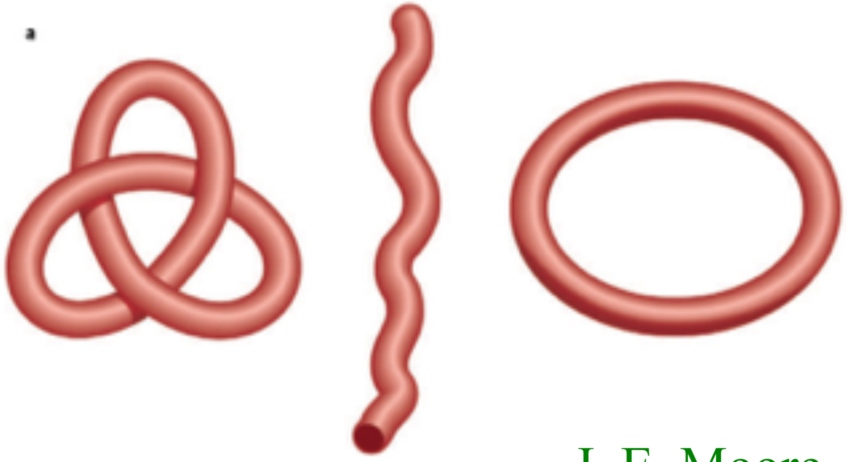




J. E. Moore, Nature (2010)

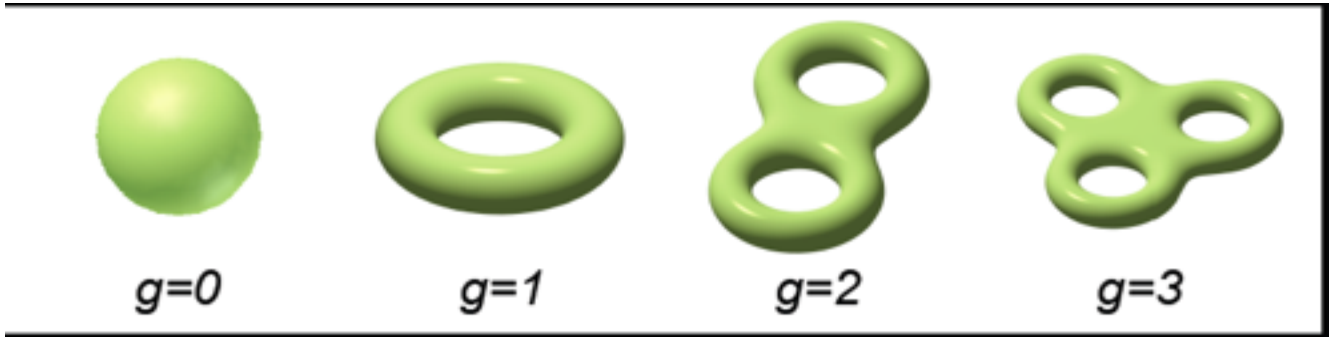
# Topological invariant

- Topological invariant： invariant under continuous transformation

Gauss-Bonnet-Chern formula:

$$\frac{1}{2\pi}\int_S K dA = 2(1-g)$$

$K$ : Gauss curvature $g$ : genus

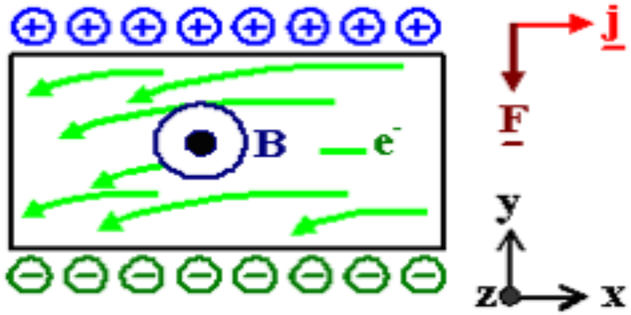



Shiing-Shen Chern
陈省身先生

- Connect the geometry to the topology.
- For 2D compact manifold, the topological invariant is the genus or Euler characteristic.

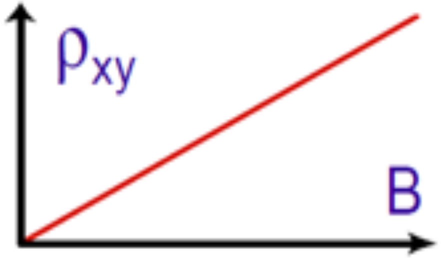
# Hall effect

Edwin Hall (1879)




**Hall resistance**

$$\rho_{xy} = \frac{E_y}{J_x} = \frac{B}{nq}$$




Charge carrier is subjected to the electric field force and Lorentz force:

$$\vec{F} = \vec{F}_e + \vec{F}_m = q(\vec{E} + \vec{v} \times \vec{B})$$

For steady state, the resultant force in y direction is zero.

$$F_y = 0 \qquad E_y = v_x B_z$$

Current density in x direction $\quad J_x = nqv_x$

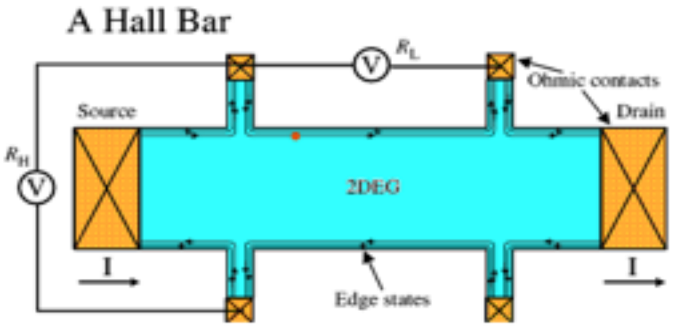
$q$ = carrier charge $\quad n$ = carrier density

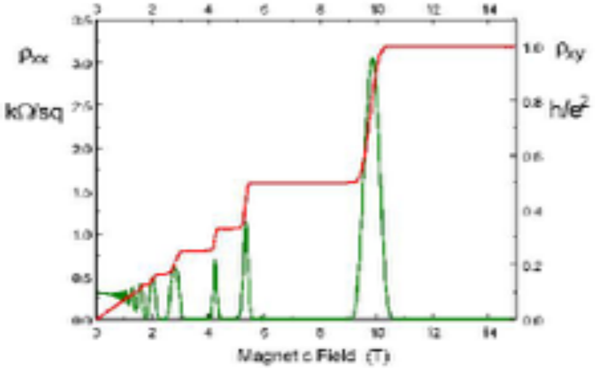
Hall resistivity $\quad \rho_{xy} = \dfrac{E_y}{J_x} = \dfrac{B}{nq}$

Application of Hall measurement:

- ❑ measure the carrier density.
- ❑ ascertain the carrier type. electron (e<0) hole (q>0)

# Quantum Hall effect




K.Von klitzing
(1980)

- **Under low temperatures and strong magnetic fields, the Hall conductance take on the quantized values.**

$$\rho_{xy} = R_Q / n \qquad \text{n = integer}$$

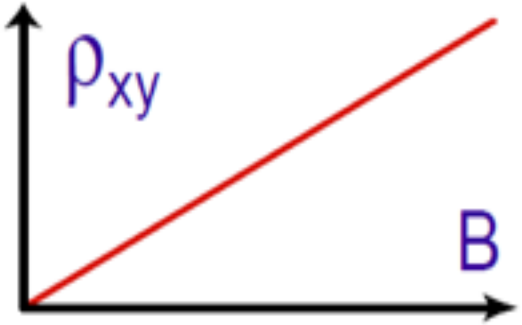
- $R_Q$ **Resistance quantum**

$$R_Q = h / e^2 = 25.812\,807 \;\; \text{k}\Omega$$

# Quantum Hall effect

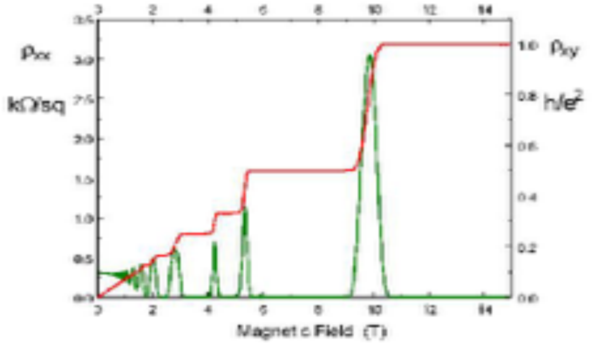
- Quantum Hall state provided the first example of a quantum state topologically distinct from all states of matter known before.

**Normal physical quantities**



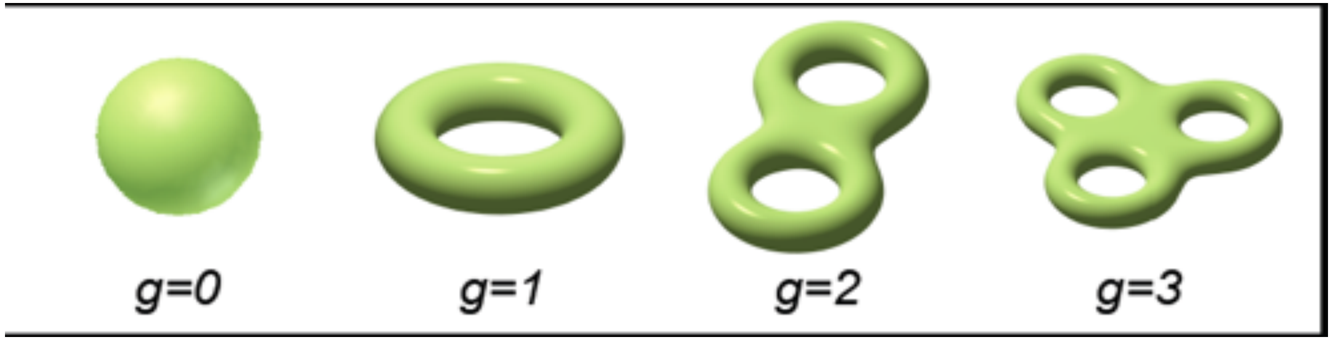

Small perturbations to the system
Small changes of physical quantities.

**Quantum Hall effect**

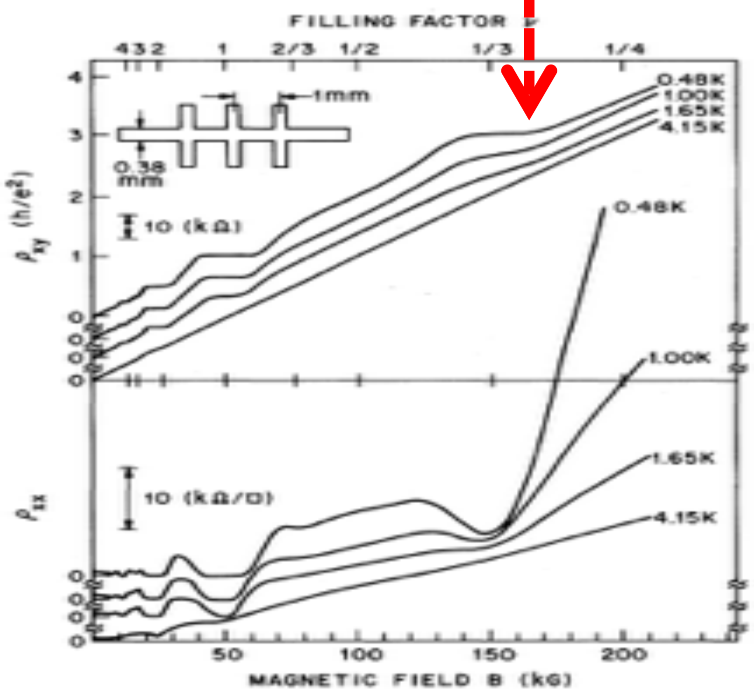



Hall resistance does not change at all if the perturbation is small

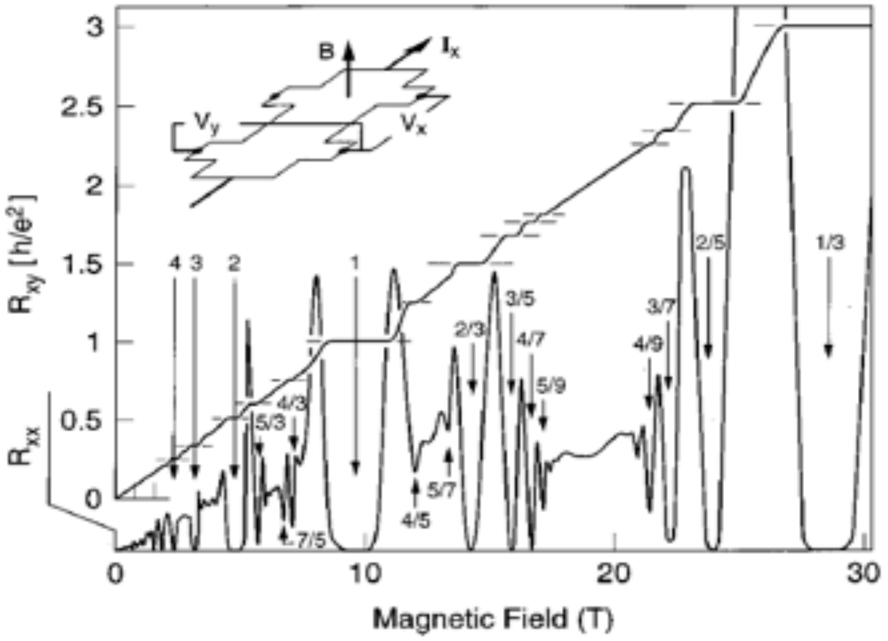
- The physical quantities are related to topological invariants



# 分数量子霍尔效应




D. C. Tsui, H. L. Stomer et al (1982)

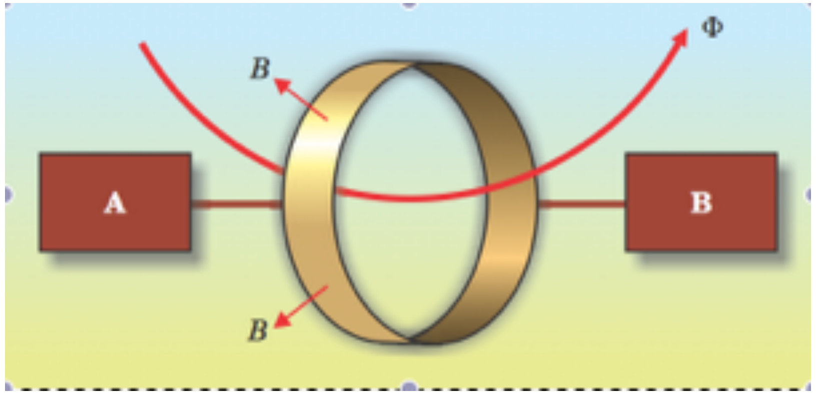



J. P. Eisenstein et al., (1990)

- ➢从整数量子霍尔效应到分数量子霍尔效应
- ❑ 更高质量的样品，更强的磁场，更低的温度
- ❑ 霍尔电阻分数量子化。 $\rho_{xy} = (p/q)R_Q$ p，q 为正整数
- ➢ 理论方面
- ❑ 电子在强磁场和相互作用下的集体行为.
- ❑ 准粒子带有分数电荷 e/q

# Topological origin of Quantum Hall effect

**Laughlin's gedanken experiment**




- **Adding** $\Delta\phi = hc/e$ **maps the system back to itself**
- **The energy increase satisfy**

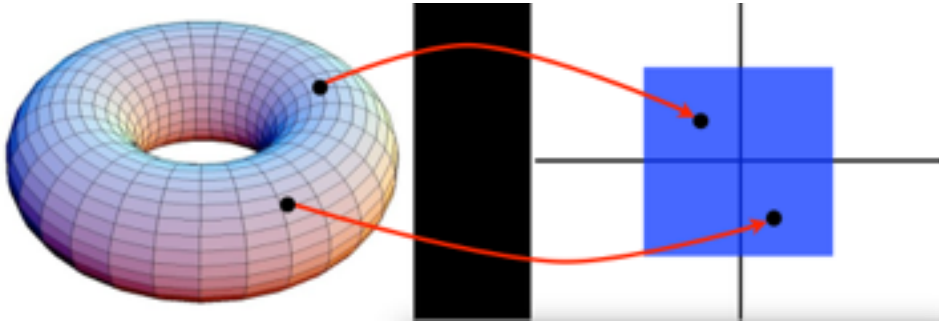
$$I = c\frac{neV}{\Delta\phi} = \frac{ne^2 V}{h}.$$

- **n is an arbitrary integer**

R. B. Laughlin, PRB (1981)

**TKNN number (Chern number)**

**Bloch Bands** **Brillouin zone**



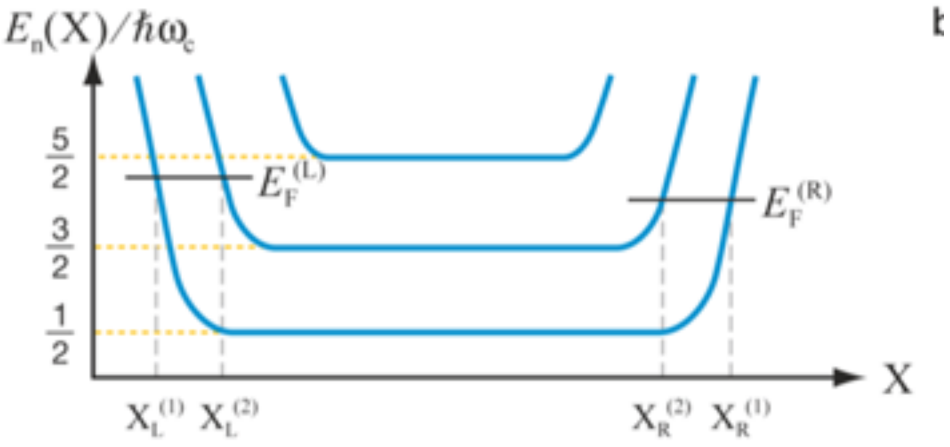
**Bloch Bands** $|\psi_n(q)\rangle = |\psi_n(q+G)\rangle$

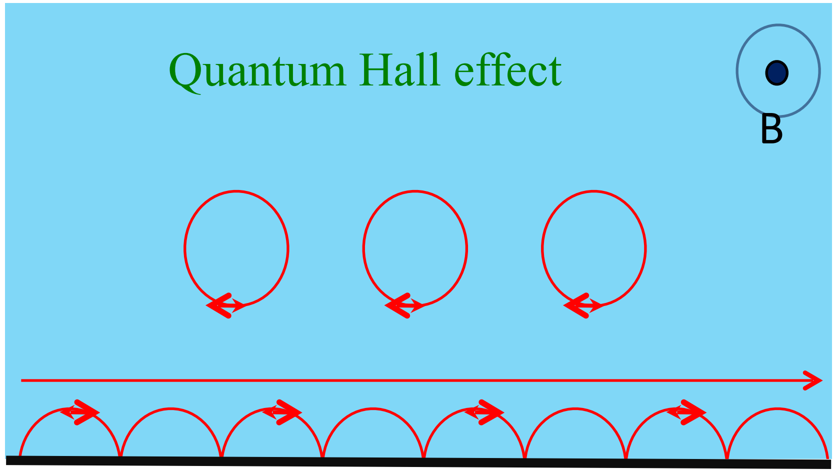
$$\boxed{\sigma_{xy} = \frac{e^2}{\hbar}\int_{BZ}\frac{d^2k}{(2\pi)^2}\Omega_{k_x k_y} = n\frac{e^2}{h}}$$

- **The Hall conductance is topological invariant (Chern number), which can only take integer values.**

TKNN, PRL (1982)

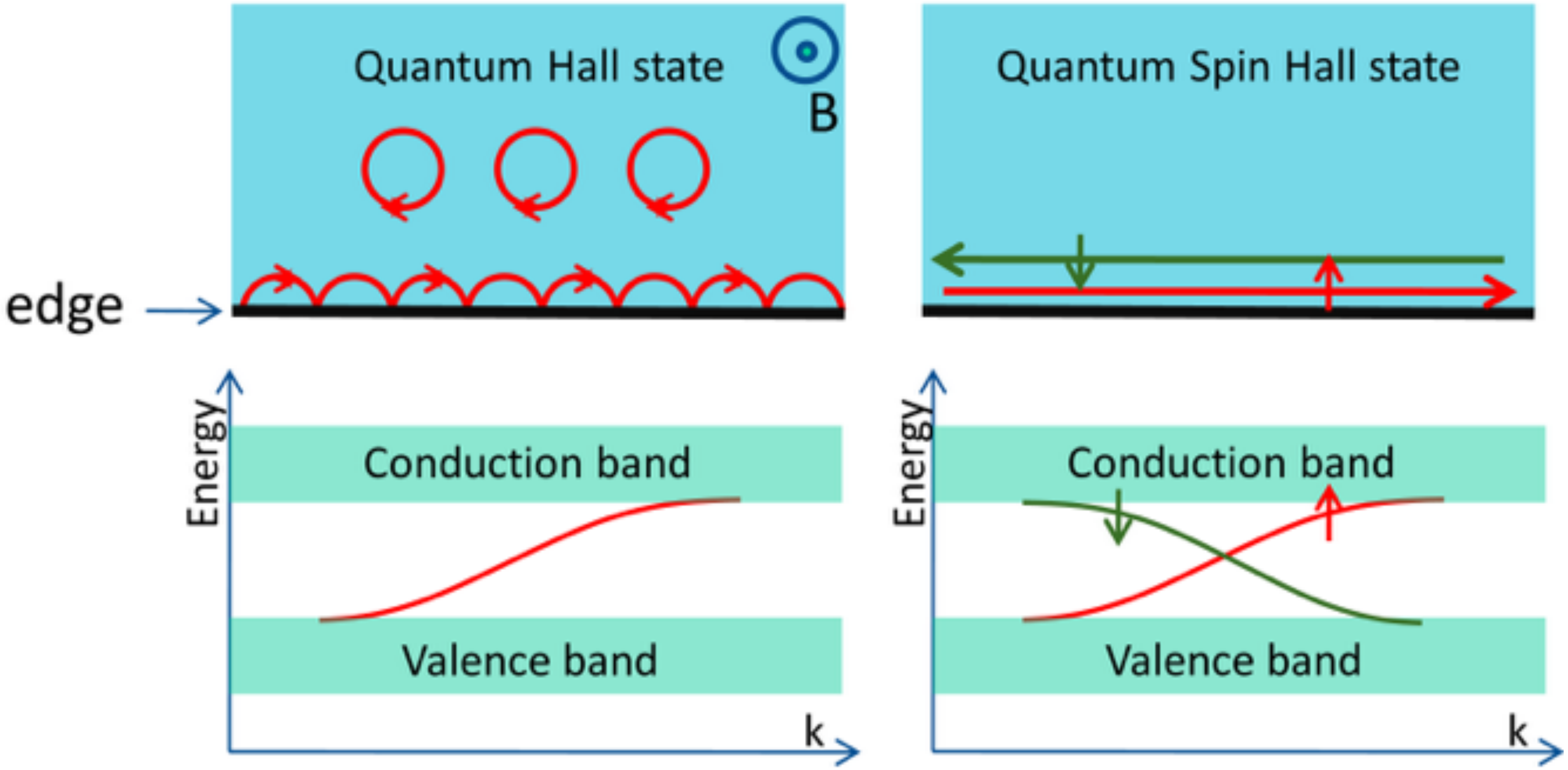
# Edge states in Quantum Hall effect




- ❑ **The bulk states form Landau Levels**
- ❑ **The bulk of the sample is insulator， but it is distinct from normal insulator. The current flow on the edge of the sample.**
- ❑ **Chiral edge transport**

# Quantum Hall effect → Quantum spin Hall effect




**Chiral edge states.**

**Break TRS with B or M.**

**No backscattering.**

**Helical edge states.**

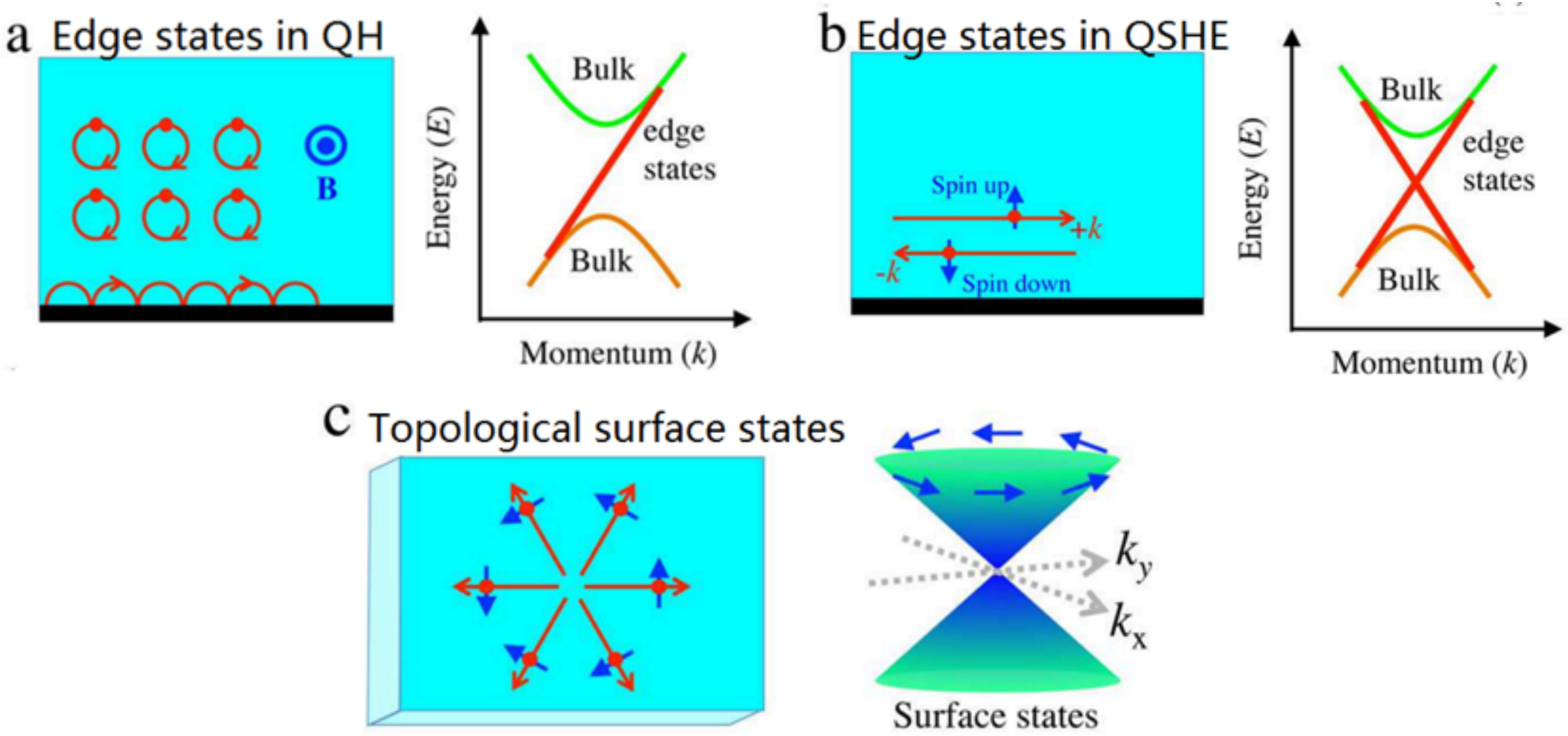
**Preserve time reversal symmetry (TRS).**

**No backscattering for TRS perturbations.**

CL Kane, EJ Mele, PRL, 95, 226801, (2005).

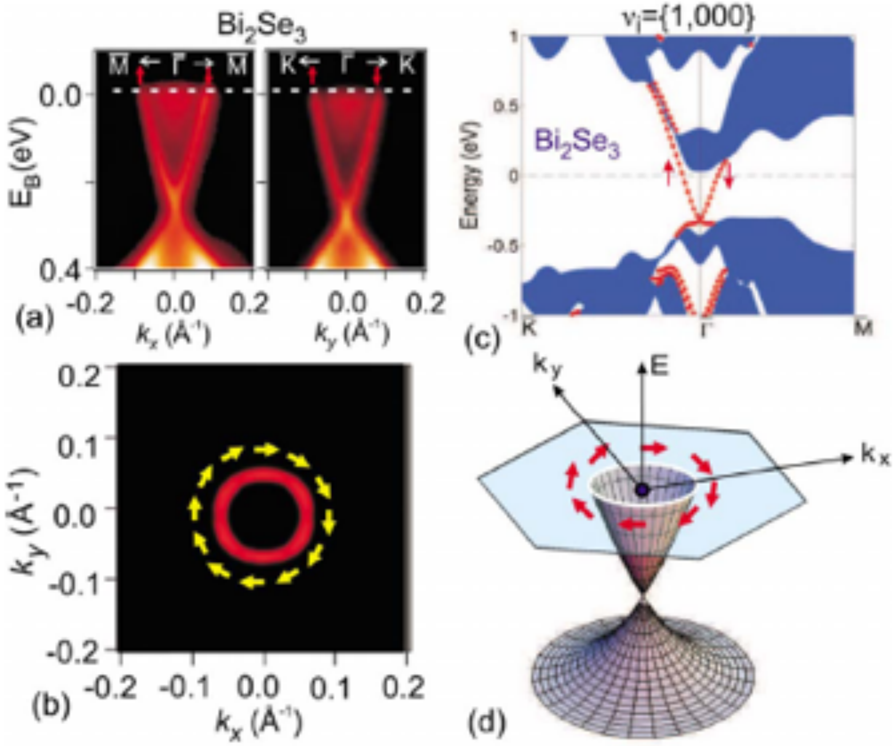
BA Bernevig, TL Hughes, SC Zhang, Science, 314, 1757 (2006).

# Quantum Hall effect → Z2 Topological Insulator

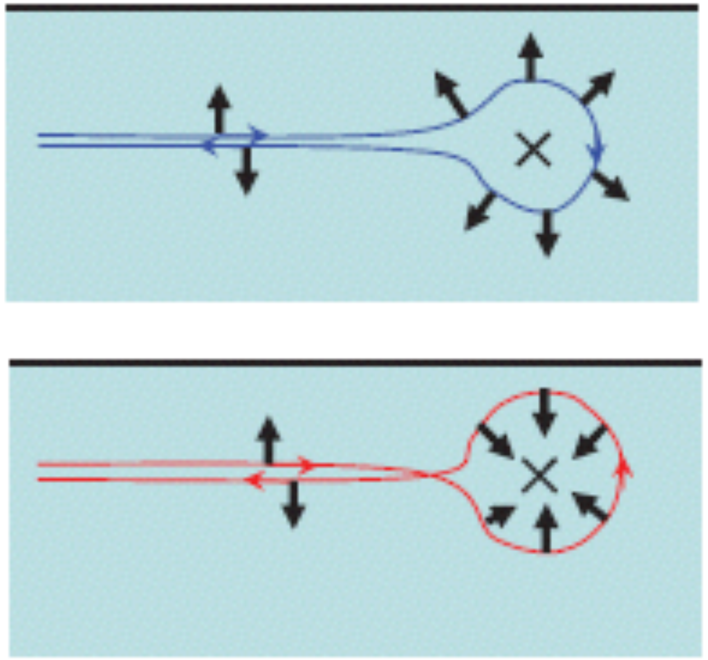



M. Z. Hasan et. al. (review) arXiv: 1406.1040v1.

# Helical surface states of 3D strong topological insulator




**Gapless surface states**

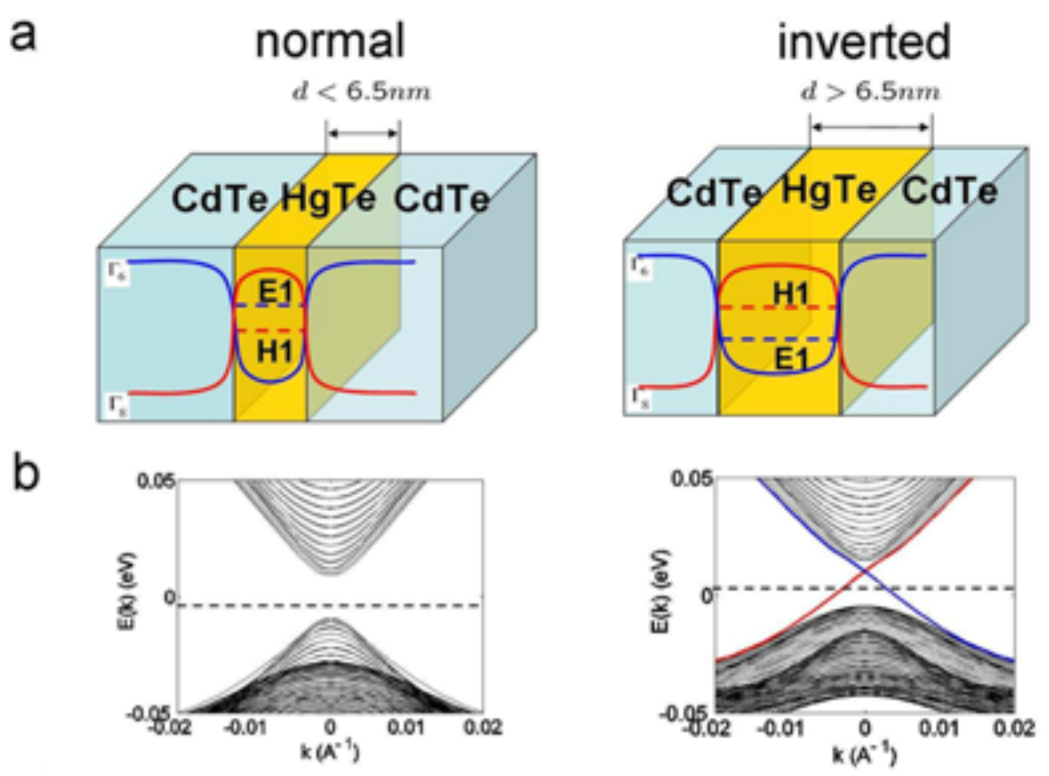


**No backscattering for nonmagnetic impurities.**

M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. 82, 3045 (2010).
X. L. Qi and S. C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).

# Realization of Quantum spin Hall effect

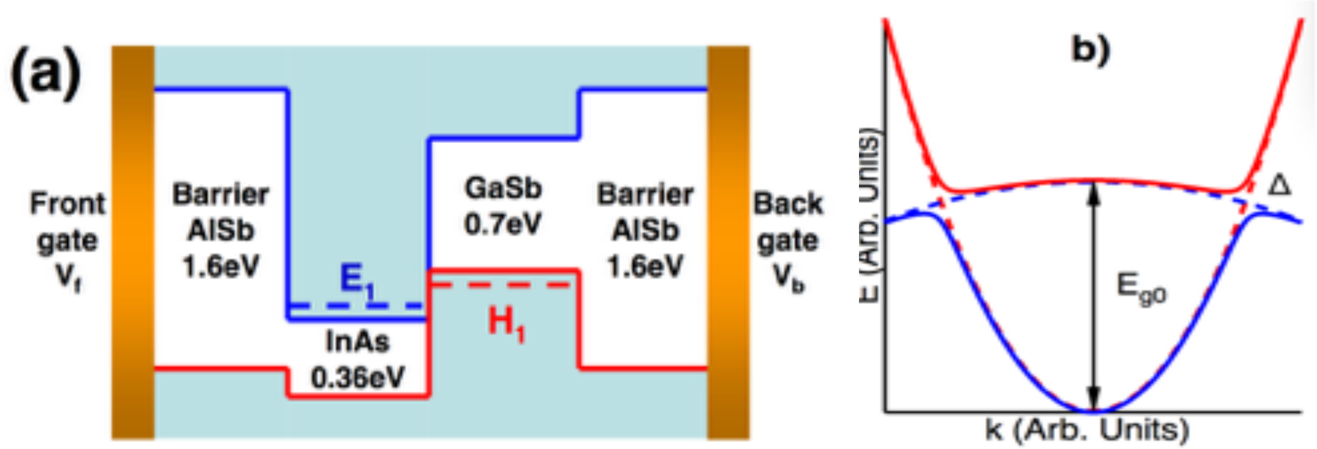
## HgTe/CdTe quantum wells



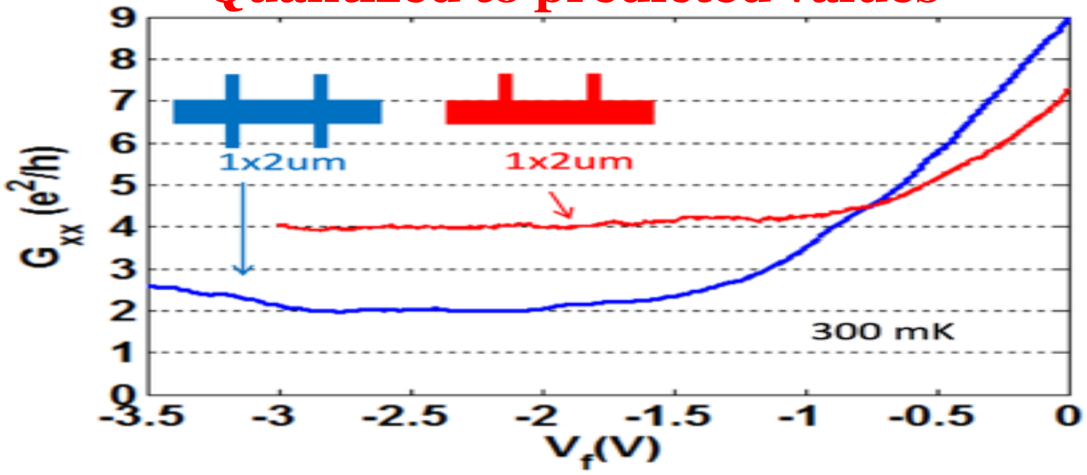

**Normal insulator** **QSHE**

B.A Bernevig et al., Science 314,1757 (2006)

## InAs/GaSb quantum wells



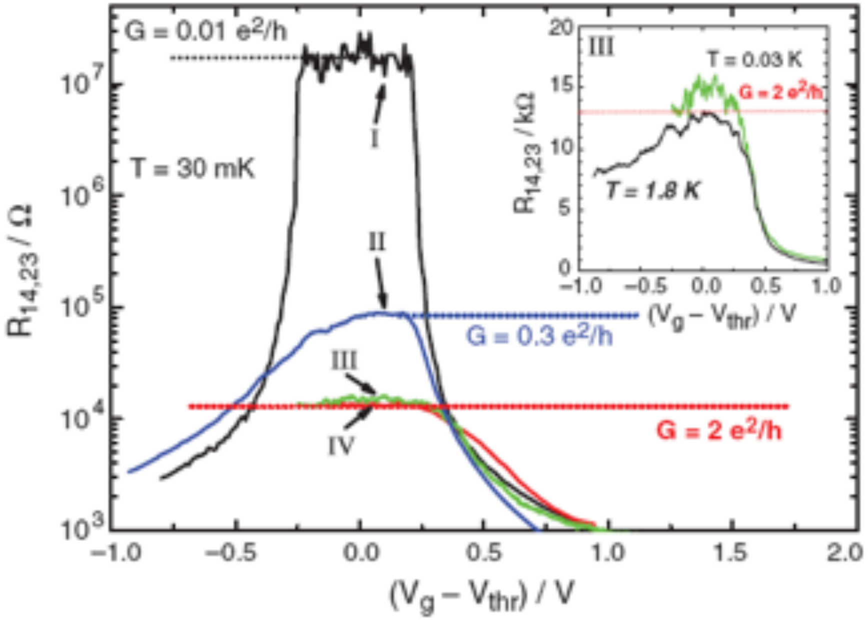

**Quantized to predicted values**

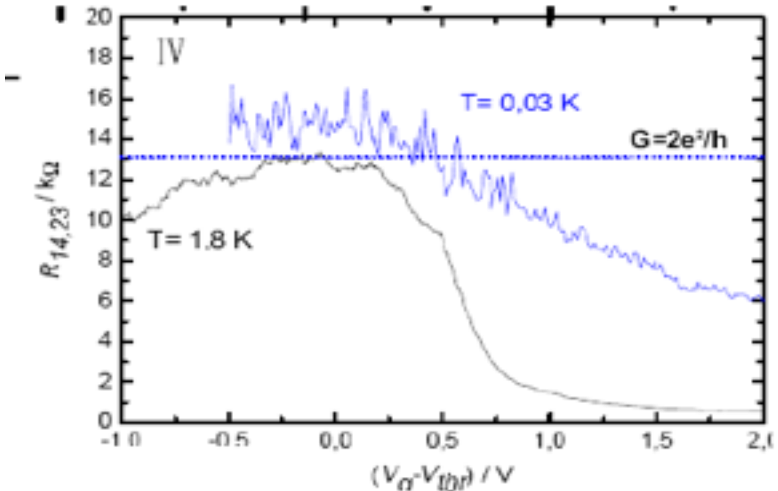



Ivan Knez et al., Phys. Rev. Lett. 107, 136603 (2011)
L. J. Du et al., arxiv:1306.1925v1 (2013)

# Transport measurement of QSHE

**HgTe/CdTe Quantum well**




➢ **d<dc normal insulator**

➢**d>dc 2D Topological insulator**

- ◻ **In small samples, R14,23 is quantized, insensitive to W variations.**
- ◻ **In large samples, R14,2 3 is no longer quantized. Why?**
- ◻ **QSH signal is robust against temperature change.**

M König, et al., Science 318, 766 (2007).

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## Dephasing effects on 2D quantum Spin hall insulators

Try to answer the questions:

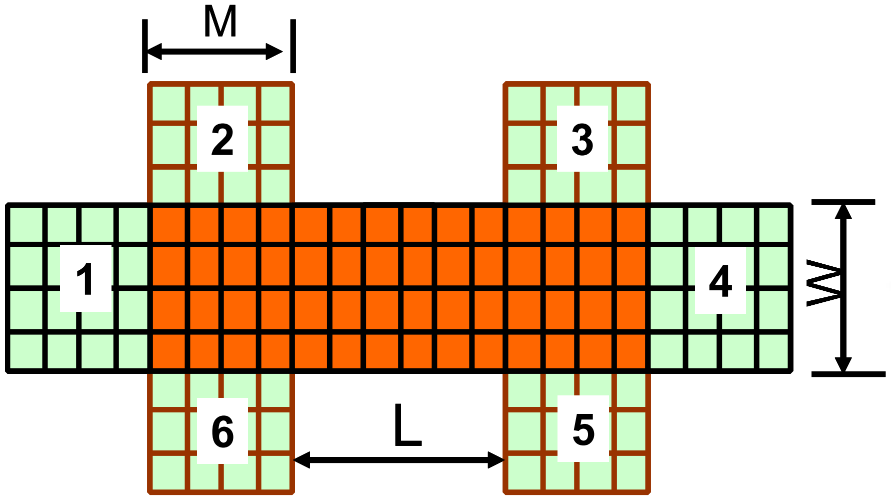
- Why longitudinal resistance only quantized in small sample ?
- Is there an observable physical quantity showing a quantized value in macroscopic samples ?

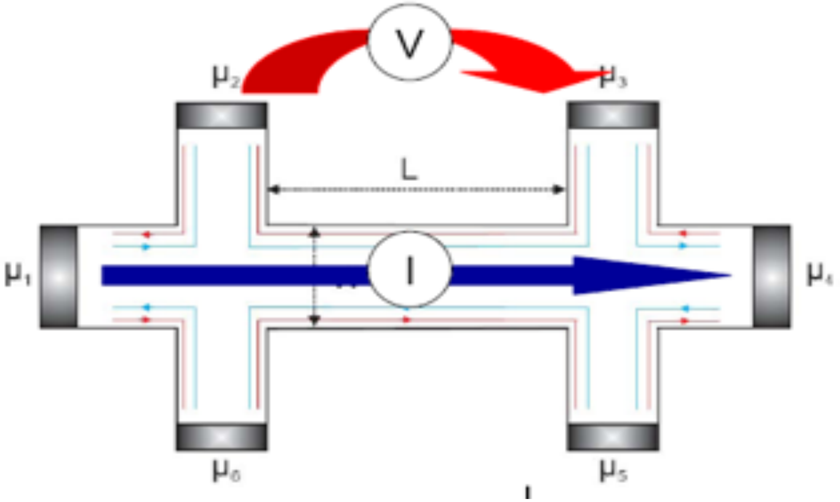
**Two types of dephasing**

- Normal dephasing: the electron-electron and electron-phonon interactions only destroy the electron phase memory. These dephasing strengths increase with rising temperature.

- Spin dephasing: Magnetic impurities, nuclear spin fluctuations and normal dephasing with strong spin-orbital interaction may destroy spin memory.

# The dephasing mode for a QSHE system




$$H = -[\sum_{i,\sigma} te^{im\sigma\phi} c^{+}_{i\sigma} c_{i+\delta x\sigma} + tc^{+}_{i\sigma} c_{i+\delta y\sigma}] + \sum_{i,k,\sigma} [\varepsilon_k a^{+}_{k\sigma} a_{Ik\sigma} + (t_k c^{+}_{i\sigma} a_{k\sigma} + t_k a^{+}_{k\sigma} c_{i\sigma})]$$

$$i = (i_x \quad i_y \qquad t \quad \hbar^2 / 2m^* a^2, \phi = qB_{eff} a^2 / \hbar$$

- The first term describes QSHE. This term can also describe QHE if the flux is spin independent.
- We use virtual leads to model dephasing processes. M. Buttiker, *Phys. Rev. B 33 3020 (1986)*

H. Jiang, S.G. Cheng, Q.F. Sun, and X.C. Xie, PRL **103**,036803(2009)

# Theoretical formalism and dephasing processes

The particle current in the lead-p (real or virtual lead) with spin $\sigma$ can be expressed

$$\boxed{I_{p\sigma} = \frac{e^2}{h} \sum_{q \neq p} T^{\sigma}_{pq} (V_{p\sigma} - V_{q\sigma})}$$

Here $V_{p\sigma}$ is the spin-dependent bias in lead p. $T^{\sigma}_{pq} = Tr[\mathbf{\Gamma_{p\sigma}} \mathbf{G^r} \mathbf{\Gamma_{q\sigma}} \mathbf{G^a}]$ is the transmission coefficient from the lead q to p with spin $\sigma$, where the linewidth functions $\Gamma_{p\sigma} = i[\mathbf{\Sigma^r_{p\sigma}} - \mathbf{\Sigma^{r+}_{p\sigma}}]$, the Green function $\mathbf{G}^r = [\mathbf{G}^a]^\dagger = [E_F \mathbf{I} - \mathbf{H}_{cen} - \sum_{p\sigma} \mathbf{\Sigma}^r_{p\sigma}]^{-1}$, $\mathbf{H}_{cen}$ is the Hamiltonian in the central region, and $\mathbf{\Sigma^r_p}$ is the retarded self-energy of lead-p that can be calculated numerically.

First type dephasing processes: normal dephasing

For each virtual lead i $\quad I_{i\uparrow} = I_{i\downarrow} \equiv 0 \qquad V_{i\uparrow} \neq V_{i\downarrow}$

The spin flip processes are forbidden.

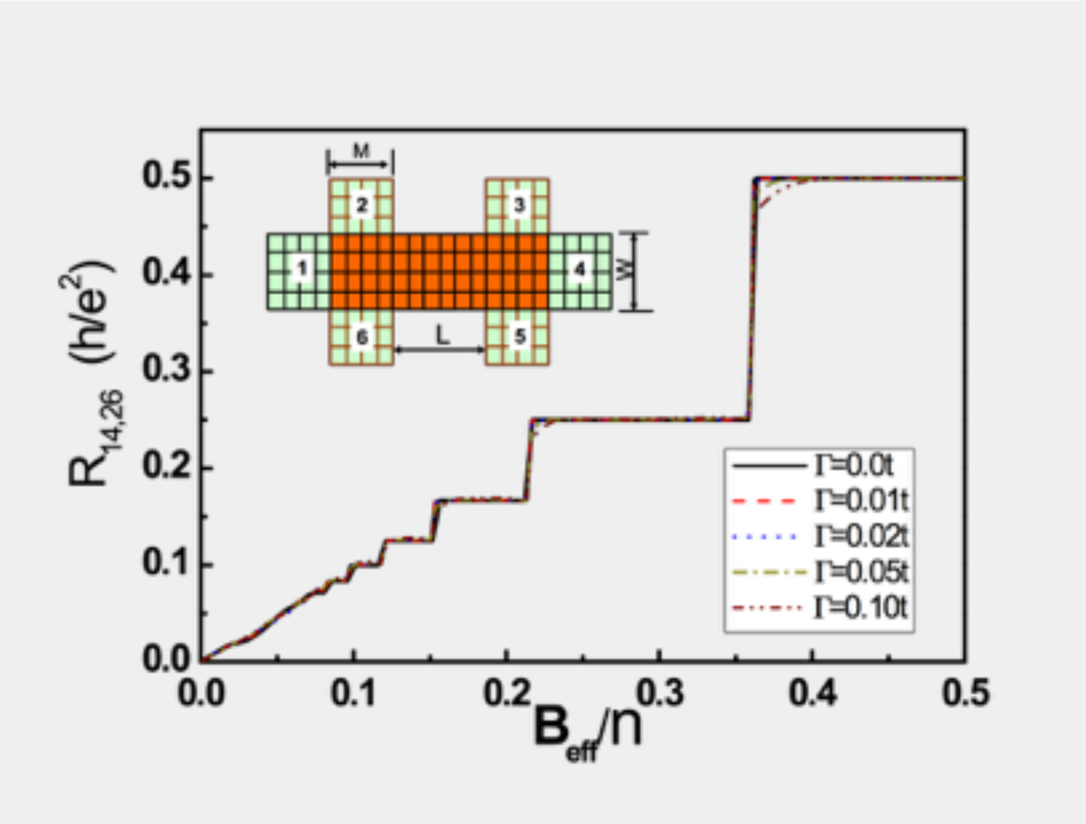
Second type dephasing processes: spin dephasing

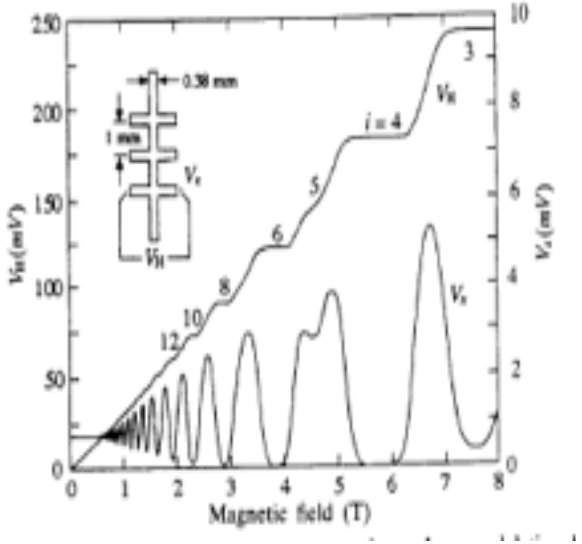
For each virtual lead i $\quad I_{i\uparrow} + I_{i\downarrow} \equiv 0 \qquad V_{i\uparrow} = V_{i\downarrow}$

The spin flip processes are allowed.

H. Jiang, S.G. Cheng, Q.F. Sun, and X.C. Xie, PRL **103**,036803(2009)

# The dephasing in QHE system

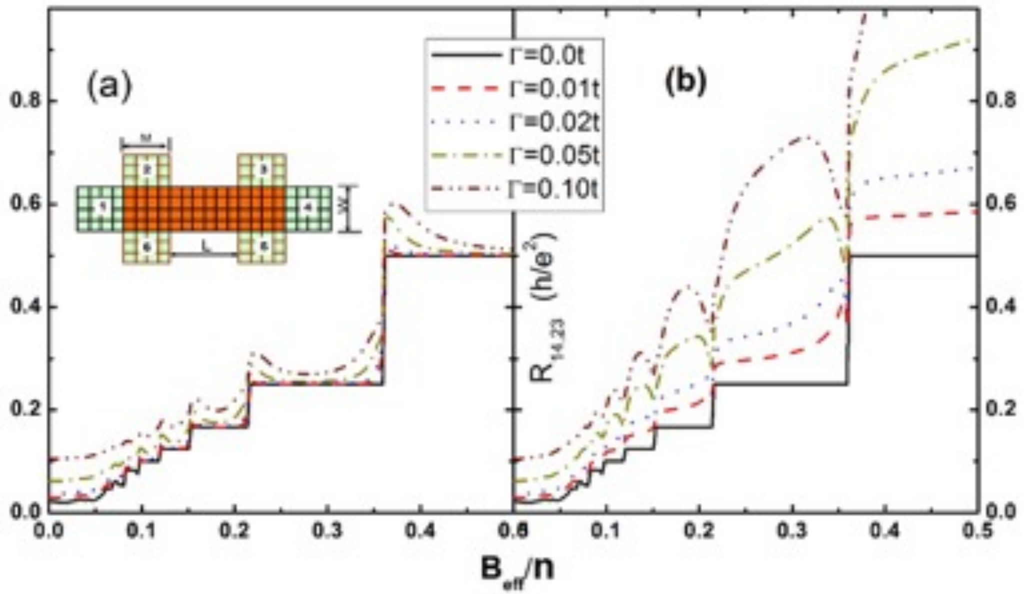



$$E_F = -3t, L = 32a, W = 32a, M = 24a$$

- The Fermi energy is fixed near the band bottom.
- The QHE is robust against dephasing.

# Dephasing in QSHE system



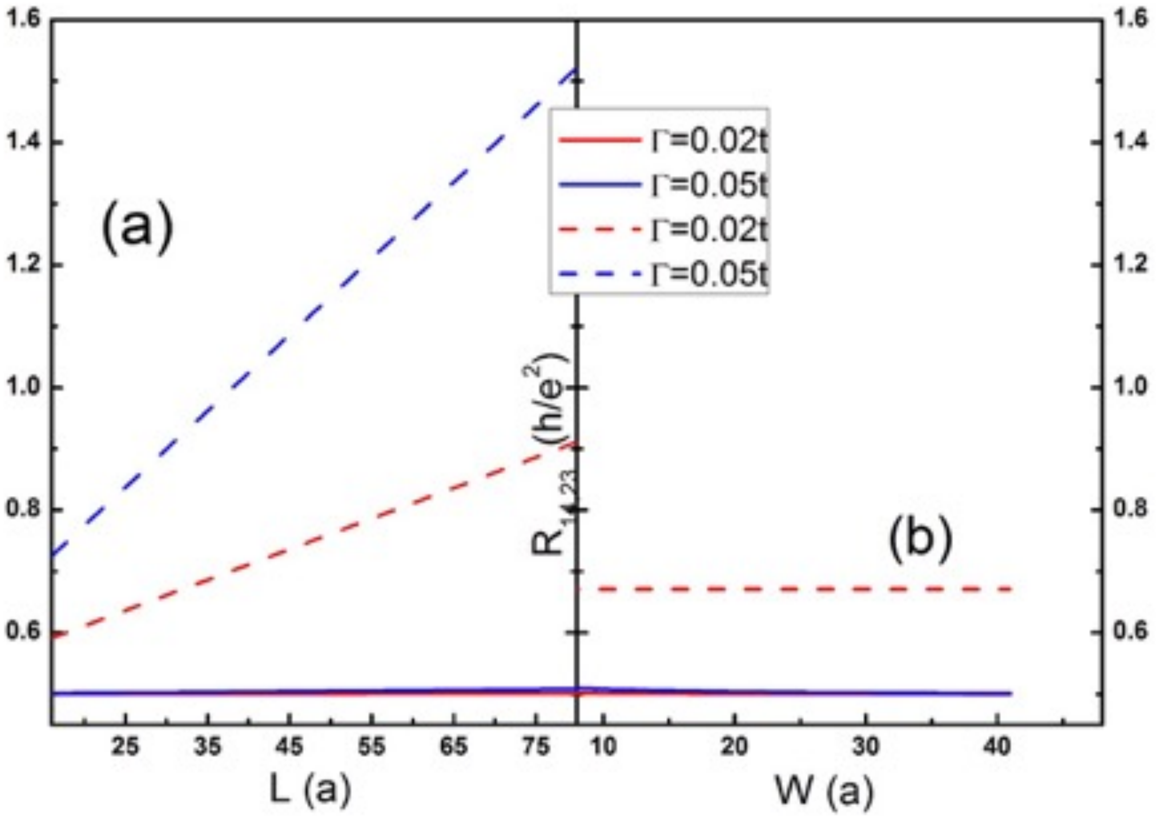

$E_F = -3t, L = 32a, W = 32a, M = 24a$

- Fig.(a) illustrates the normal dephasing while Fig.(b) is for the spin dephasing.
- In the low field, the result is consistent to that of the semi-classical Drude model.
- The dephasing strength is characterized by $\Gamma$

**The transport property is robust to normal dephasing, but fragile to spin dephasing.**

H. Jiang, S.G. Cheng, Q.F. Sun, and X.C. Xie, PRL **103**,036803(2009)

# Dephasing in QSHE system: II




•The dashed line denotes the spin dephasing process, and the solid line denotes the normal dephasing process.

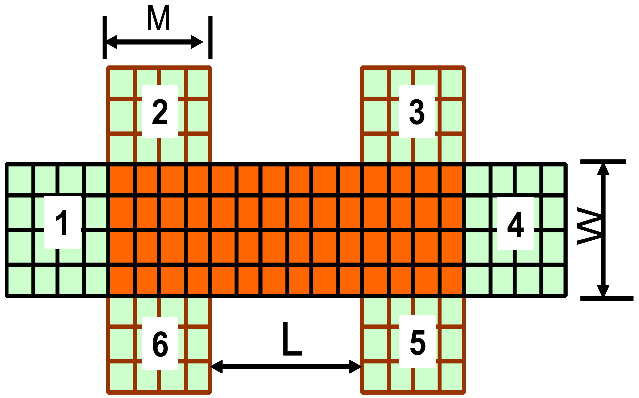
$$E_F = -3t, M = 24a$$

$$B_{\text{eff}} = 0.5, W = 32a....(a)\, B_{\text{eff}} = 0.3, L = 32a....(b)$$

**The QSH signal is insensitive to width variation but sensitive to the length change.**

H. Jiang, S.G. Cheng, Q.F. Sun, and X.C. Xie, PRL **103**,036803(2009)

# Spin accumulations: formula and reasoning



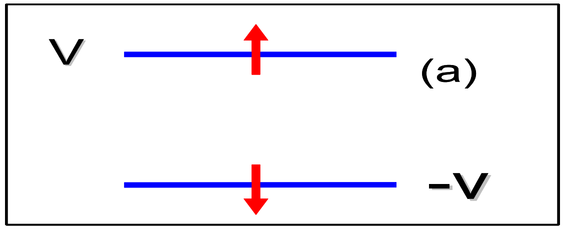

Longitudinal leads $\quad V_{1\uparrow} = V_{1\downarrow} = -V_{4\uparrow} = -V_{4\downarrow} = V$

Traverse leads $\quad I_{p\uparrow} \equiv 0, I_{p\downarrow} \equiv 0, V_{p\uparrow} \neq V_{p\downarrow}$

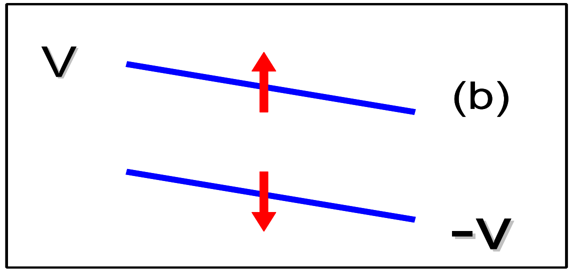
resistance $\quad R_{\mathrm{p,s}} = \dfrac{V_{p\uparrow} - V_{p\downarrow}}{I_1}$

for a given edge

in the absence of dephasing




in the present of dephasing

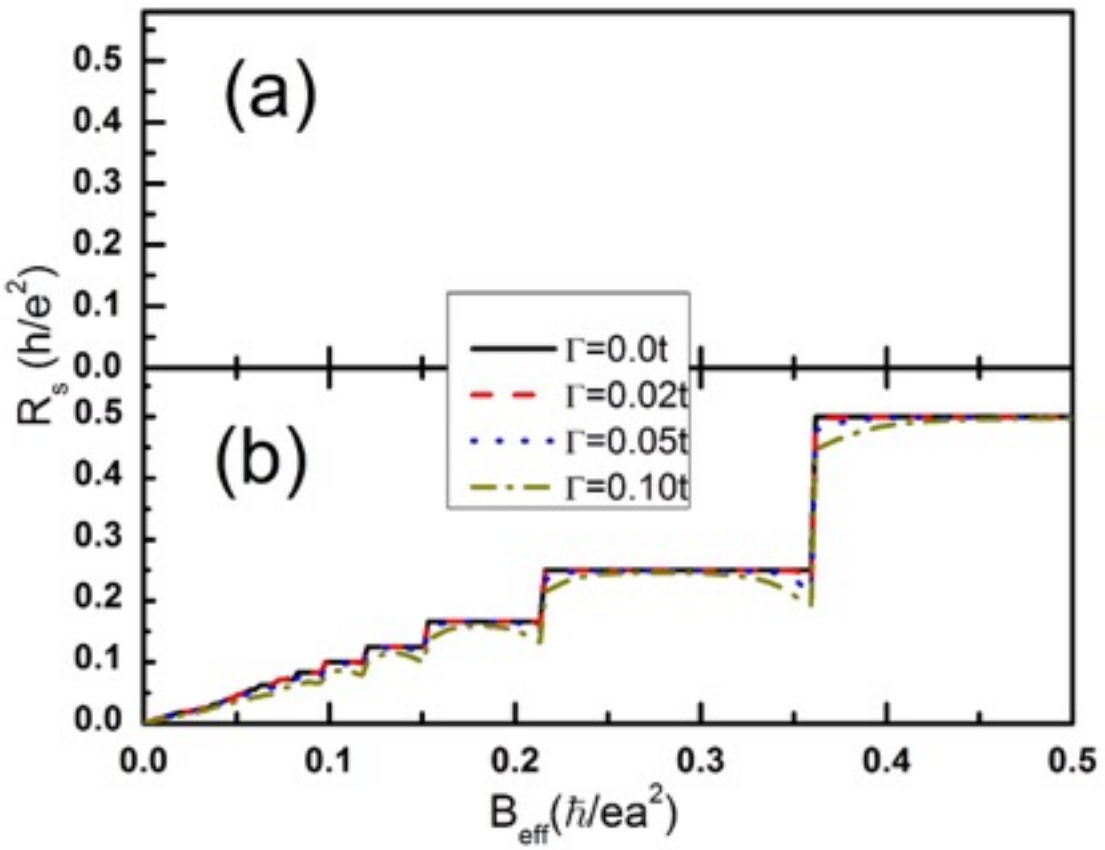



spin flip $\quad V_{p\uparrow} - V_{p\downarrow}$

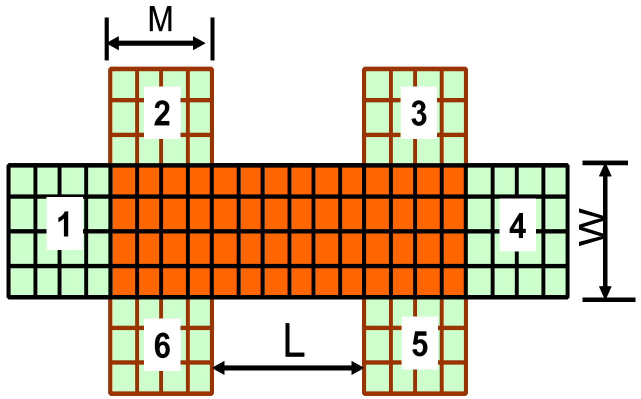
$I_1$

H. Jiang, S.G. Cheng, Q.F. Sun, and X.C. Xie, PRL **103**,036803(2009)

# Dephasing in QSHE system




$E_F = -3t, M = 24a, W = 32a, L = 32a$




$$R_s = (V_{2\uparrow} - V_{2\downarrow})/I_{14}$$

Rs is robust against any dephasing processes

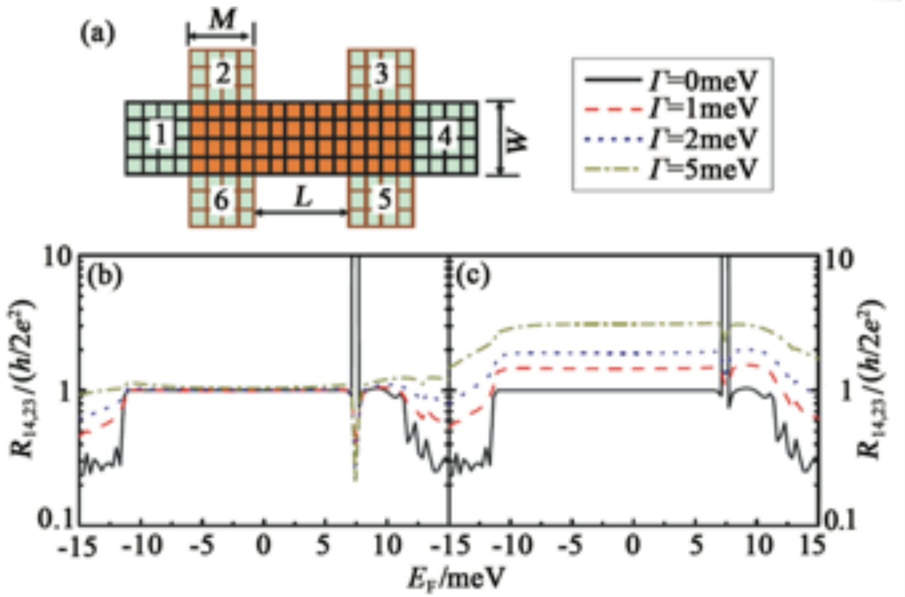
$$R_{2,s} = R_{3,s} = -R_{5,s} = -R_{6,s}$$

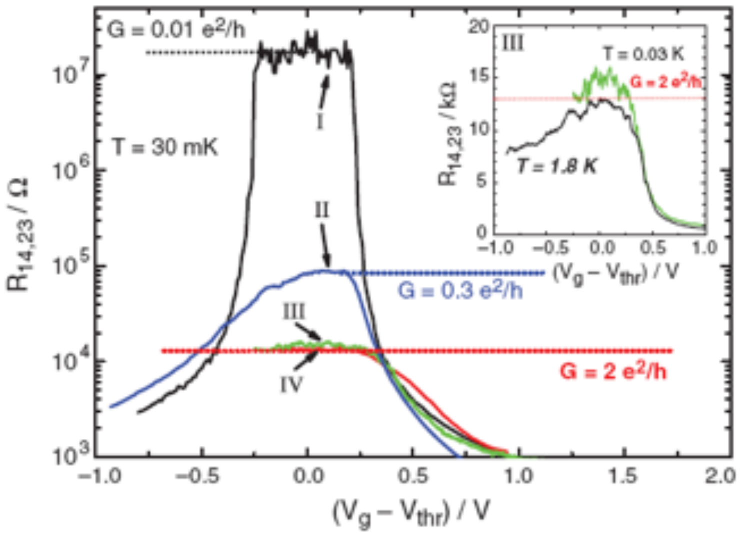
H. Jiang, S.G. Cheng, Q.F. Sun, and X.C. Xie, PRL **103**,036803(2009)

# Dephasing in QSHE system IV: HgTe/CdTe QWs

$$H = H_{\mathrm{QSHE}} + H_{\mathrm{virtual-lead}} + H_{\mathrm{couple}}$$

$H_{\mathrm{QSHE}}$: **HgTe/CdTe QWs model**




**Normal dephasing** **Spin dephasing** **Experiments**

- **The longitudinal resistance of HgTe/CdTe QWs is insensitive to normal dephasing but sensitive to the spin dephasing.**
- **Numerical calculation is consistent with the experimental data.**


H. Jiang, S.G. Cheng, Q.F. Sun, and X.C. Xie, PRL, 103, 036803 (2009).

H. Jiang, S.G. Cheng, Q.F. Sun, and X.C. Xie, Physics, 40, 454 (2011).

# Conclusion on dephasing effect in 2D quantum Spin Hall insulators

- The spin dephasing plays important roles in QSHE, it makes the quantized longitudinal resistance only observable in mesoscopic systems. However, the spin accumulation measurement that is robust against any dephasing may provide a new playground.

- As far as helical edge state is not destroyed, the spin Hall resistance is quantized, independent of model or material detail. Therefore, it can better reflect the topological nature of QSHE. The spin Hall resistance can be measured by measurements of the polarization resistance.

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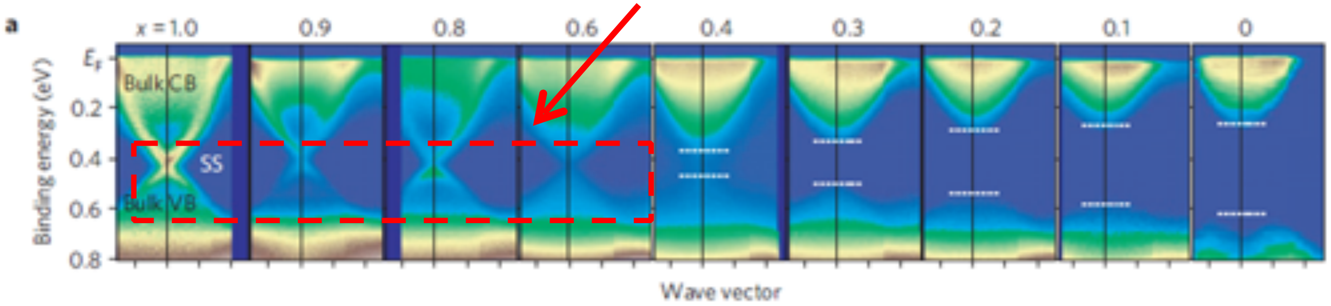
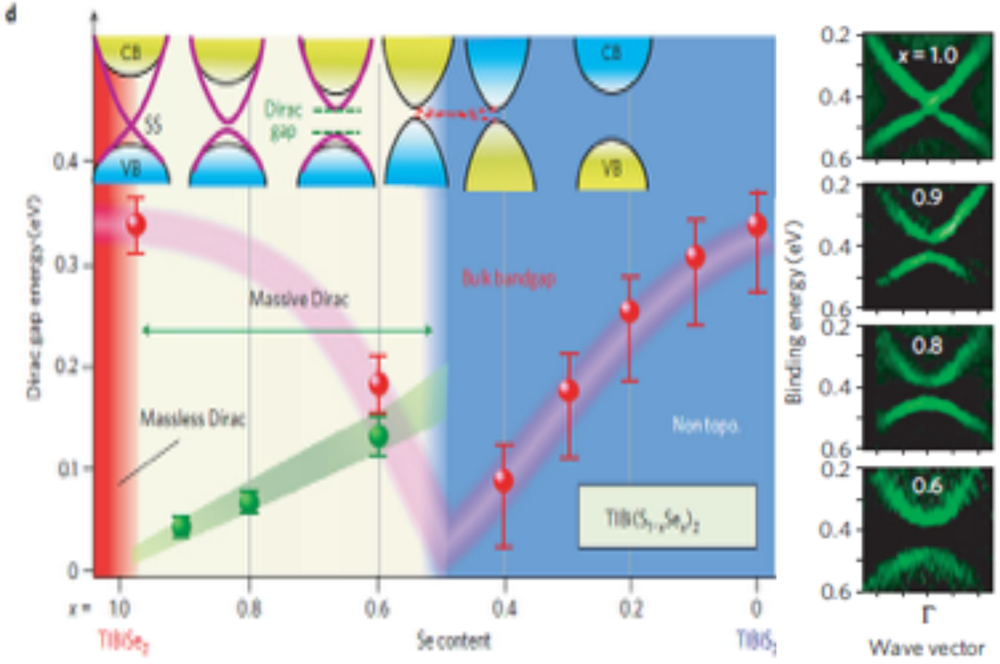
# Anomalous ‘gap-like’ features in $TlBi(S_{1-x}Se_x)_2$ surface states

**Anomalous ‘gap-like’ features around the Dirac point**




- **TQPT in a tuneable spin orbital system.**
- **The TQPT occur at critical point δ = 0.5.**




- **The surface states acquire ‘gap-like’ feature before the TQPT point.**
- **The surface state preserve the TRS.**
- **The typical gap is 50 meV for x=0.9.**

T. Sato, et al., Nature Physics 7, 840 (2011); Su-Yang Xu et al., arXiv: 1206.0278.

# Anomalous ‘gap-like’ features in $TlBi(S_{1-x}Se_x)_2$ surface states

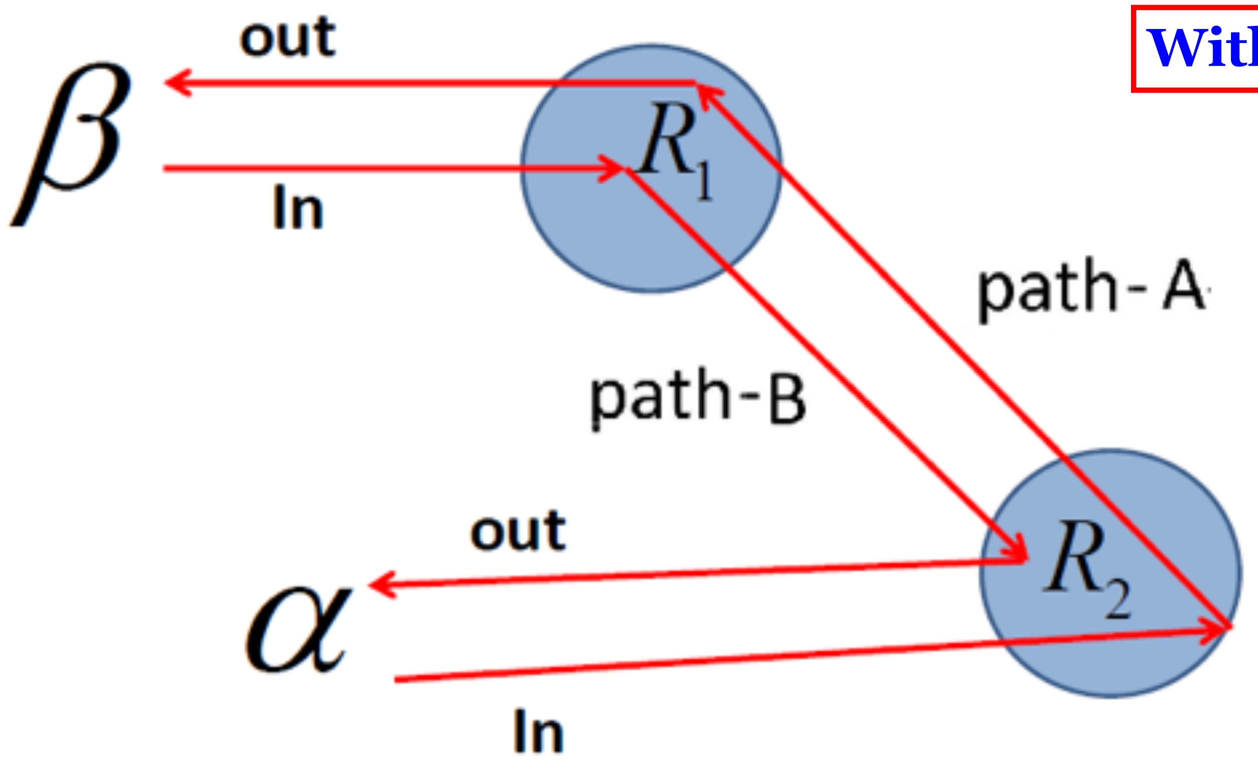
**What mechanism leads to the anomolous ‘gap-like’ feature in helical surface states?**

- ❑ in the strong 3D topological insulator
- ❑ preserve the spin-momentum locking (helical) property
- ❑ no magnetic impurity, thus no spin dephasing
- ❑ large sample thickness, thus no finite size effect

**Re-examine the normal dephasing effect.**

## The role of dephasing effect on backscattering

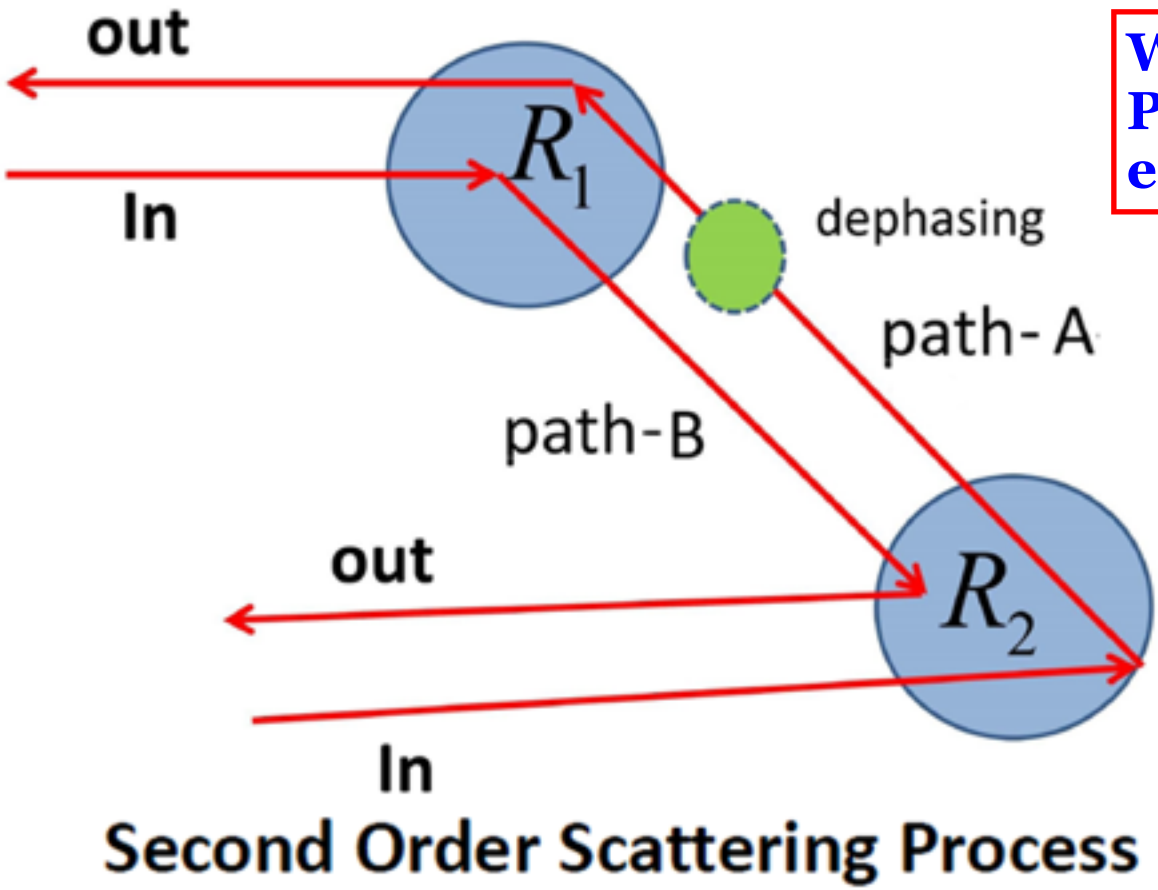
**Without dephasing**




Second Order Scattering Process

**The π Berry phase between A-path and B-path eliminates the backscattering.**

# The role of dephasing effect on backscattering




**With normal dephasing: Phase uncertainty due to e-e and e-ph interactions**

**The phase uncertainty between A-path and B-path leads to backscattering.**

# Scattering process of helical surface states

**Surface states Hamiltonian:**

$$H(\mathbf{k}) = s\hbar v_f (\hat{\mathbf{z}} \times \mathbf{k}) \cdot \sigma$$

**The scattered wave function:**

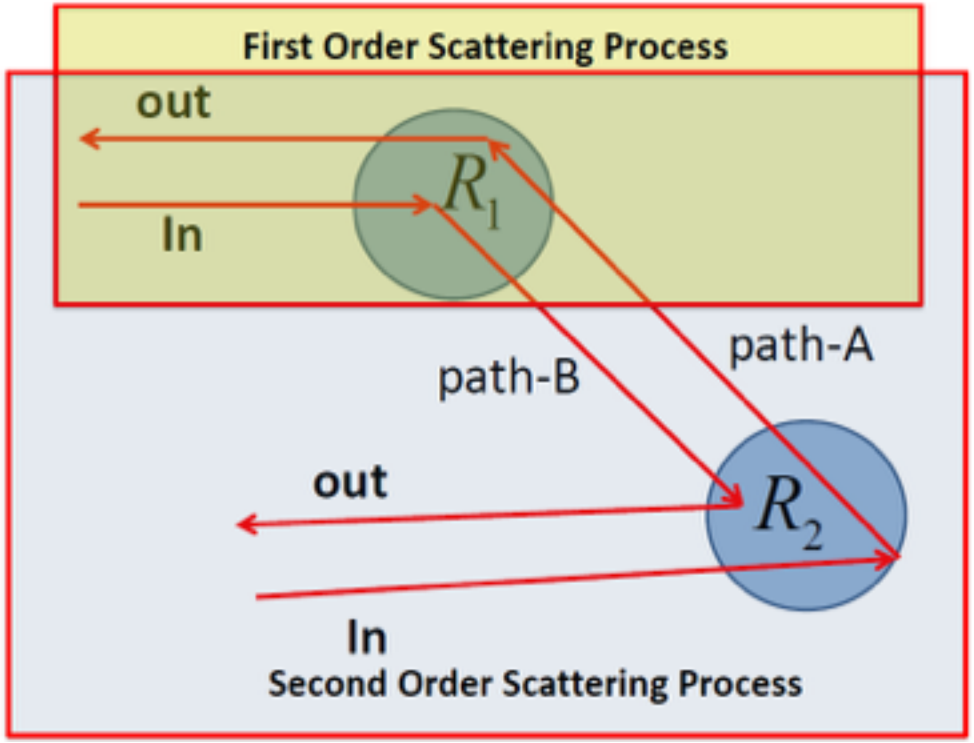
$$\psi = \varphi_{\vec{k}_{in}}(\vec{r}) + \frac{f(\theta_{in}, \theta_{out})}{\sqrt{r}} \begin{pmatrix} 1 & e^{i\theta_{out}} \end{pmatrix}^T e^{ikr}$$

**The scattering amplitude:** $f(\theta_{in}, \theta_{out})$

**The scattering cross section:** $\sigma(\theta_{in}, \theta_{out}) = |f(\theta_{in}, \theta_{out})|^2$

**The scattering process is determined by the scattering cross section** $\sigma(\theta_{in}, \theta_{out})$.

# The backscattering amplitude: without dephasing




**The scattering amplitude:**

**First order:**

$$f_1(\theta_{in}, \theta_{out}) = \frac{1}{v_f}\sqrt{\frac{2k}{-i\pi}}\left(1 + e^{-i(\theta_{out}-\theta_{in})}\right)\Gamma$$

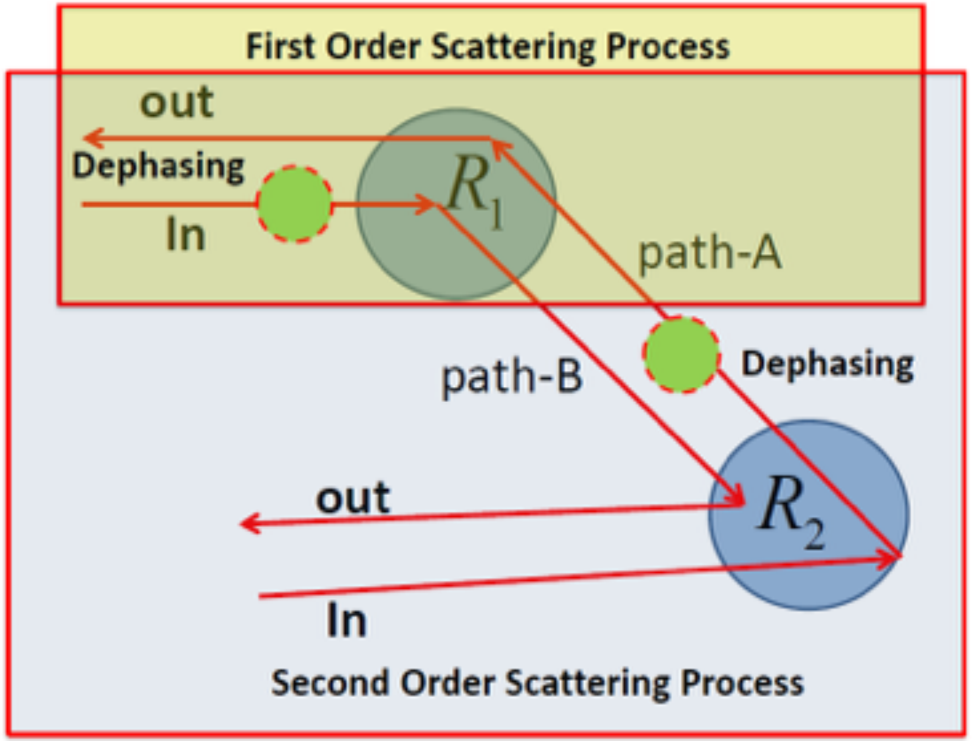
**Second order:**

$$f_2(\theta_{in}, \theta_{out}) = \frac{4k\left(1 + e^{-i(\theta_{out}-\theta_{in})}\right)}{-i\pi v_f^2\sqrt{|\mathbf{R}_2 - \mathbf{R}_1|}}\Gamma^2$$

□. **No Backscattering without dephasing because of destructive interference between A-path and B-path.**

H.W. Liu, H. Jiang, Q.F. Sun, and X.C. Xie, PRL 113, 046805 (2014)

# The backscattering amplitude: with normal dephasing




**The backscattering cross-section:**

**First order:**

$$\sigma_1(\theta_{out} = \pi + \theta_{in}, \theta_{in}) = 0$$

**Second order:**

$$\sigma(\pi + \theta_{in}, \theta_{in}) = \frac{8k^2\Gamma^4 \langle \delta\varphi^2 \rangle}{\pi^2 v_f^4 |\mathbf{R_2} - \mathbf{R_1}|}$$

**Phase uncertainty:** $\langle \delta\varphi^2 \rangle$
**Scatter strength:** $\Gamma$
**Impurity distance:** $|R_2 - R_1|$

**□. Normal dephasing causes backscattering at the second order process, while the first order backscattering cross section remains zero.**

# Backscattering cross-section: Anderson impurity and charge impurity

**Anderson impurity:** $U_S(\mathbf{r} - R_i) = U_0 \delta(\mathbf{r} - \mathbf{R_i})$

**Backscattering cross-section:**

$$\sigma_S(\pi + \theta_{in}, \theta_{in}) = \frac{8k^2 U_0^4 \langle \delta\varphi^2 \rangle}{\pi^2 v_f^4 |\mathbf{R_2} - \mathbf{R_1}|} \propto k^2$$

**Dominant at high energy**

**Charge impurity:** $U_L(\mathbf{r} - R_i) = \frac{\hbar v_f \Delta}{|\mathbf{r} - R_i|}$

**Backscattering cross-section:**

**Dominant at low energy**

$$\sigma_L(\pi + \theta_{in}, \theta_{in}) = \frac{8\Delta^4 \langle \delta\varphi^2 \rangle}{\pi^2 k^2 |\mathbf{R_2} - \mathbf{R_1}|} \propto 1/k^2$$

- **Combination of dephasing and charge impurity cause extremely large backscattering around the Dirac point.**

# Combined effect of dephasing and charge impurity scattering

**Band broadening effect of surface quasi-particle:**

$$-2\, Im \sum^{R}(k,\omega) = \frac{\hbar}{\tau} = \hbar v_f n_i \sigma_T$$

**The imaginary part of quasi-particle self energy:**

$$\boxed{-Im\, \Sigma^{R}(k,\epsilon_k) = \frac{2n_i\xi^2}{\epsilon_k} + \frac{4n_i\xi^4}{\hbar\pi v_f^2 \tau_\varphi \epsilon_k^2}}$$

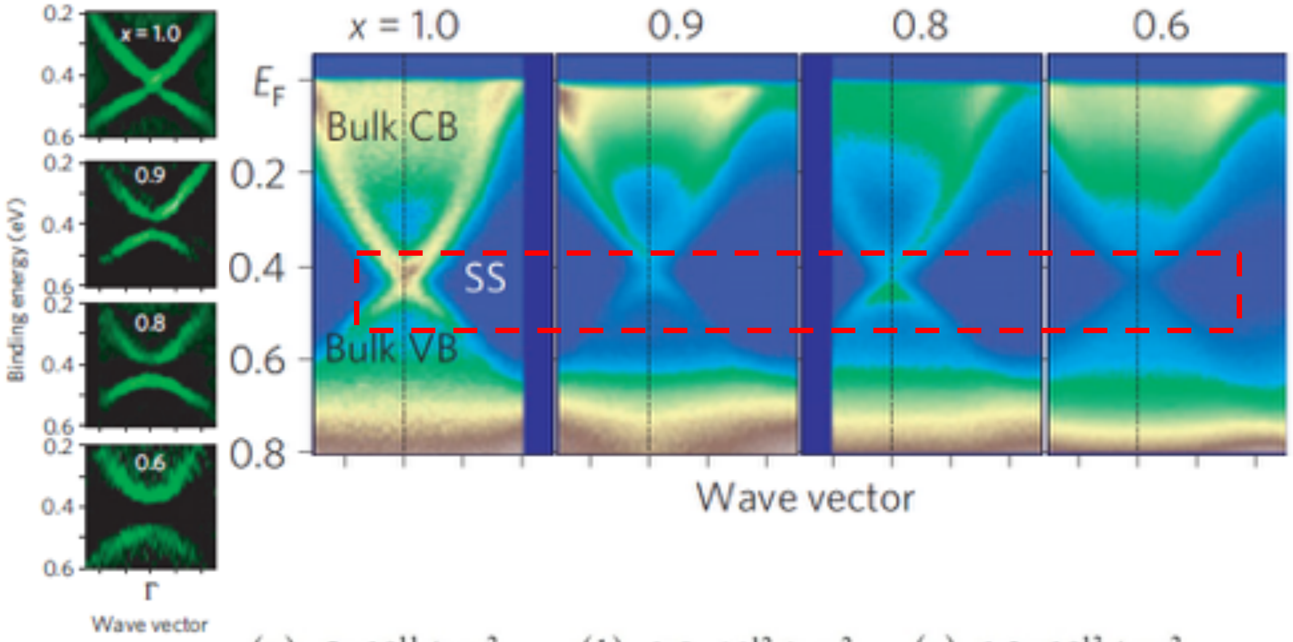
**Total transport cross section:** $\sigma_T$

**Charge impurity concentration:** $n_i$

**Dephasing time:** $\tau_\varphi \simeq 10^{-10}s$

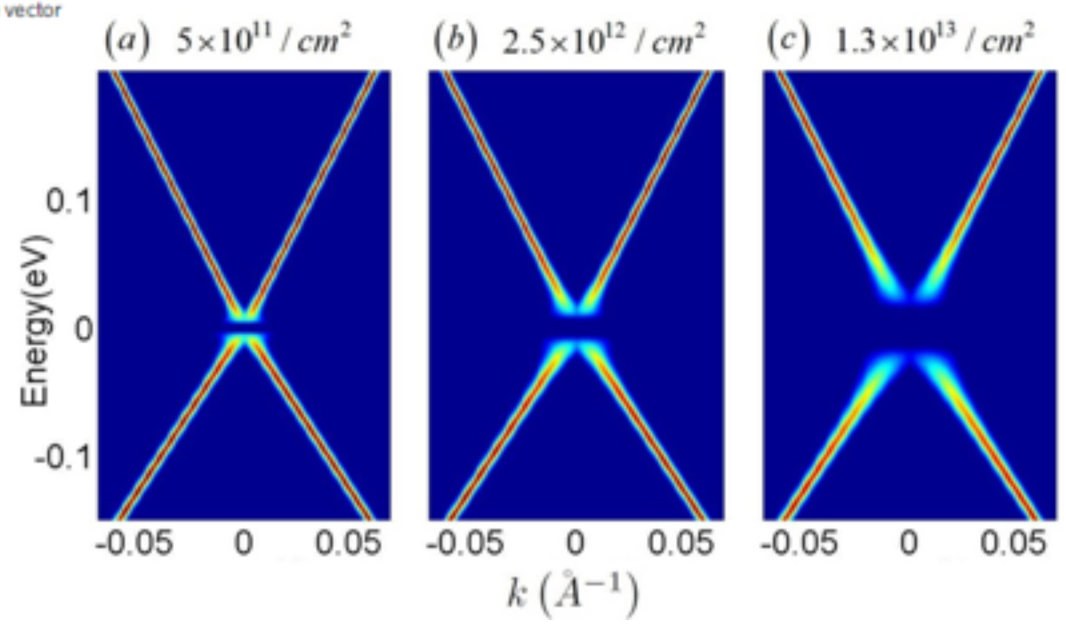
**Temperature dependent**

# Comparison with experimental results




Experiment:
T. Sato, et al., Nature Physics 7, 840 (2011).

The large ratio of S substitution for Se (>10%) may lead to substantial crystal vacancies, and these vacancies play the role of charge impurity.




Simulation:
Bandwidth broadening effect.

## Conclusion on dephasing effect in 3D TIs

- 3D TIs surface states:

  - The combined effect of normal dephasing and impurity scattering can cause backscattering in the helical SS.

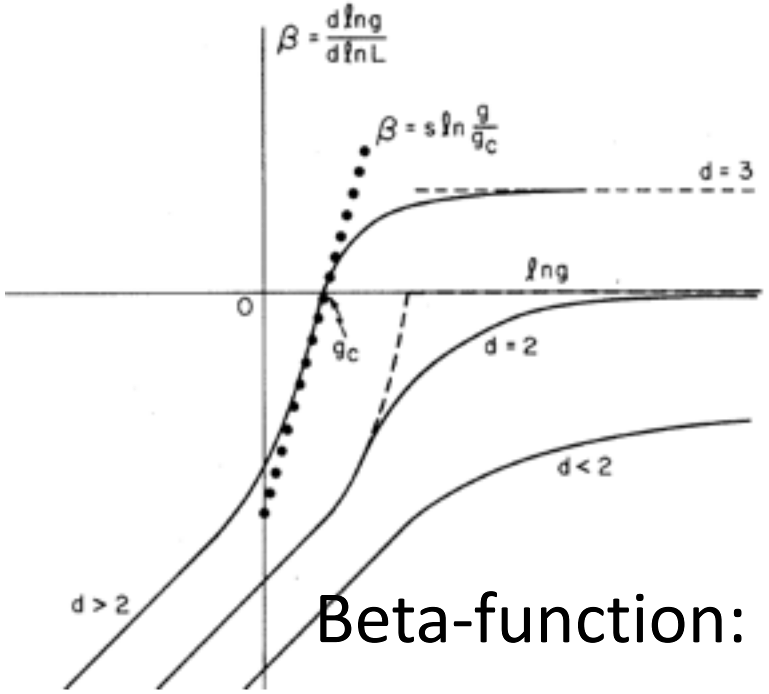
  - Normal dephasing and charge impurity cause large backscattering around the Dirac point, and leads to the anomalous 'gap-like' features found in recent experiments.

# Outline

- Introduction
  - From quantum Hall systems to topological insulators
- Dephasing effects in topological insulators
  - Edge states of 2D quantum spin Hall insulators
  - Surface states of 3D topological insulators
- Disorder effects in topological insulators
  - Tunable Anderson transition in 2D quantum spin Hall insulators
  - Multiple Anderson transition in Weyl semimetals

All the states are localized in 1D and 2D without:
(i) magnetic field; (ii) SOC;(iii)electron-electron interaction

E. Abrahams, et al, PRL. 42, 673 (1979).




Beta-function:

$$d \ln g(L)/d \ln L = \beta(g(L)).$$

**Anderson transition in 2D and symmetry class**

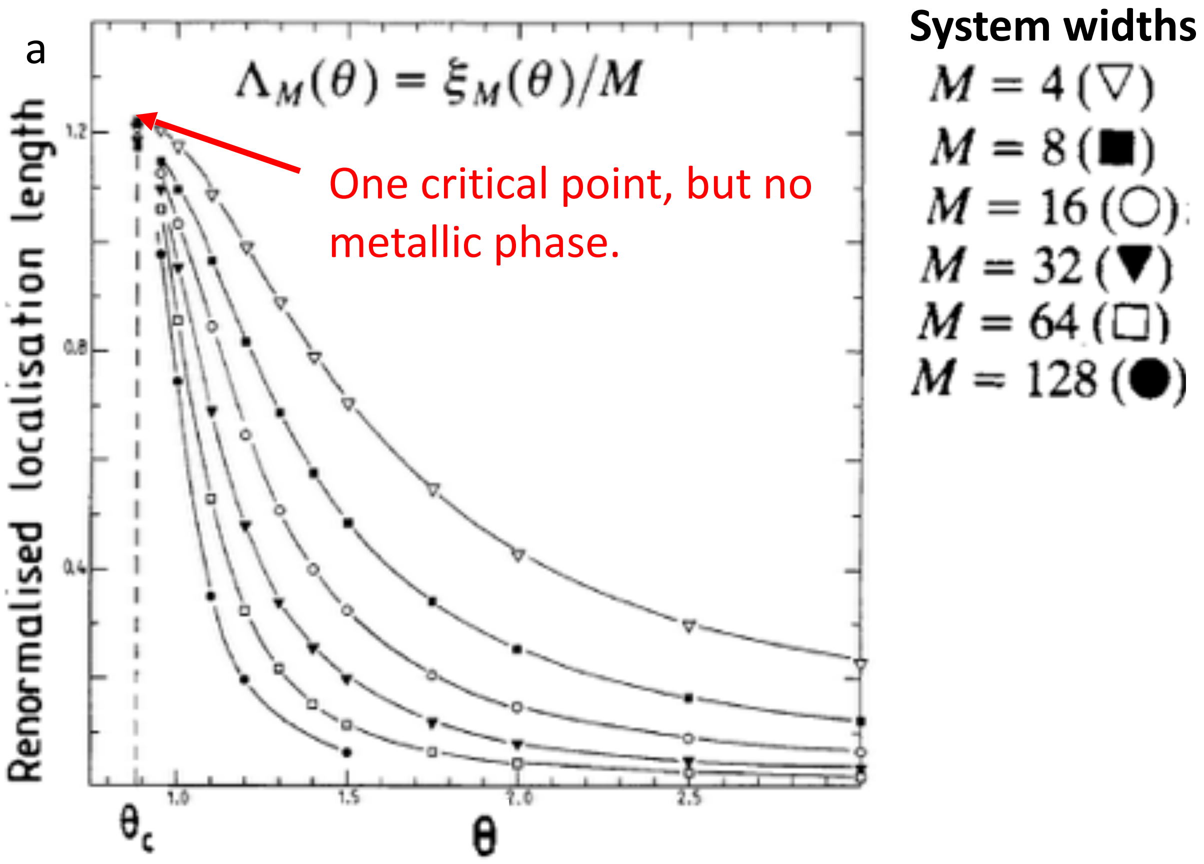
- **Orthogonal : TRS and spin rotation symmetry**
  - **all the states localized**
- **Unitary : break TRS**
  - **e.g. QHE and QAH, critical point**

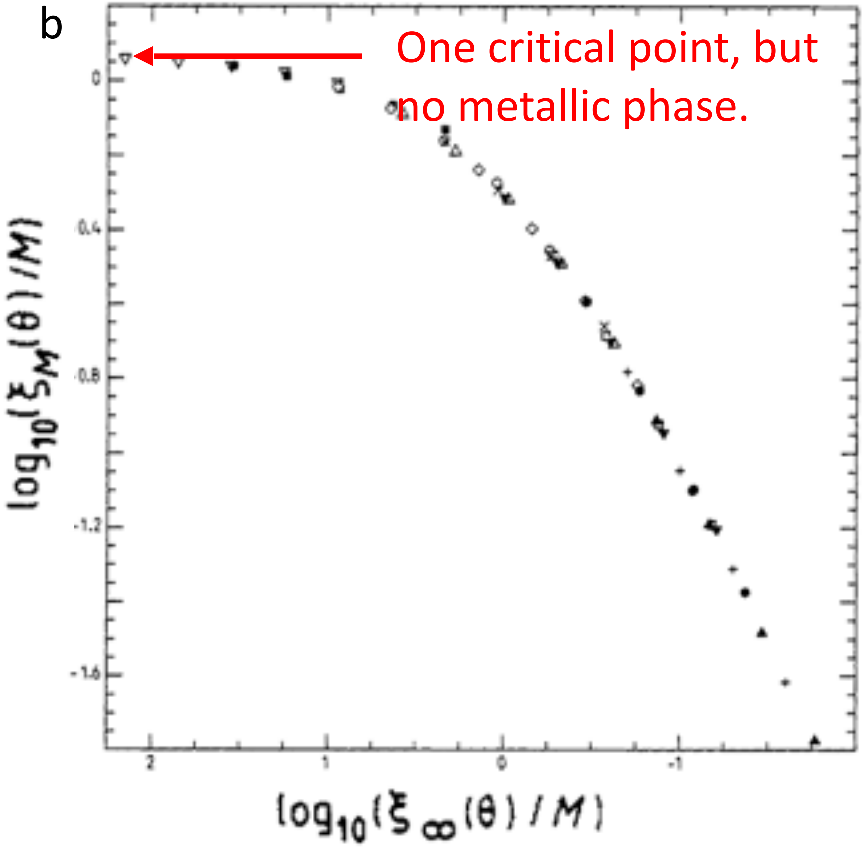
Possible Metallic Phase?

- **symplectic : TRS without spin rotation symmetry**
  - **SOC, metallic phase**

A. Huckestein, RMP. 67, 357 (1995).
F. Evers et al., RMP,80, 1355 (2008)

# Scaling in IQHE (Unitary class): critical point but no metallic phase.




- The renormalized localization length of integer quantum Hall effect as a function of energy. The vertical break line marks the Landau level centre.
- The one parameter scaling of the renormalized localization length in integer quantum Hall effect.

JT Chalker and PD Coddington J. Phys. C. 21, 2665 (1988).

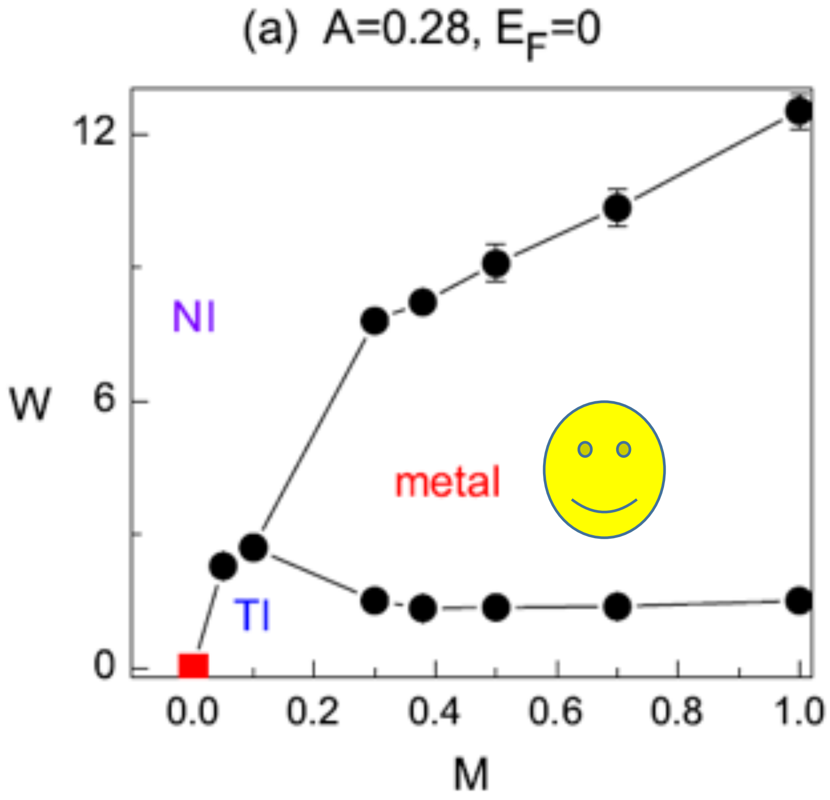
# Tunable Anderson transition (AT) in QSH insulator

**Spin-up BHZ Hamiltonian** (unitary)

$$H_0(k) = A(k_x\tau_x + k_y\tau_y) + (M - Bk^2)\tau_z + V_{dis}\tau_0$$

- **Tunable parameters**
  - **topological mass *M***
  - **Electron-hole hybrid strength *A***
  - **Fermi energy $E_F$**

- **Metallic phase in 2D unitary class**
- **Anderson transitions are dependent on the parameter M**
- **TI-Metallic-NI at large M**



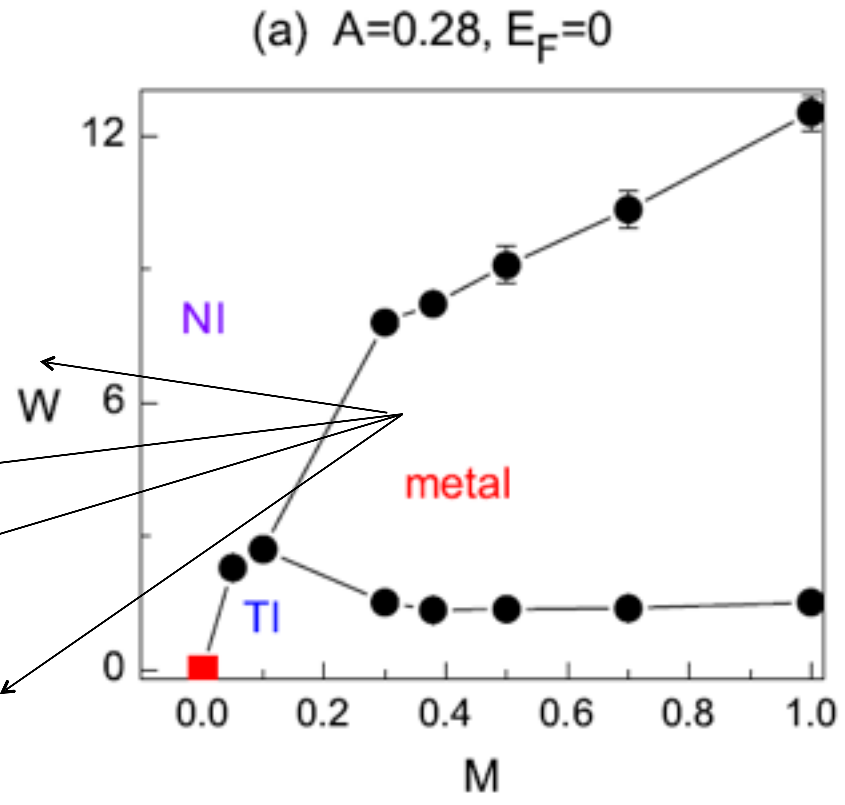
# Tunable Anderson transition (AT) in QSH insulator

## Spin-up BHZ Hamiltonian (unitary)

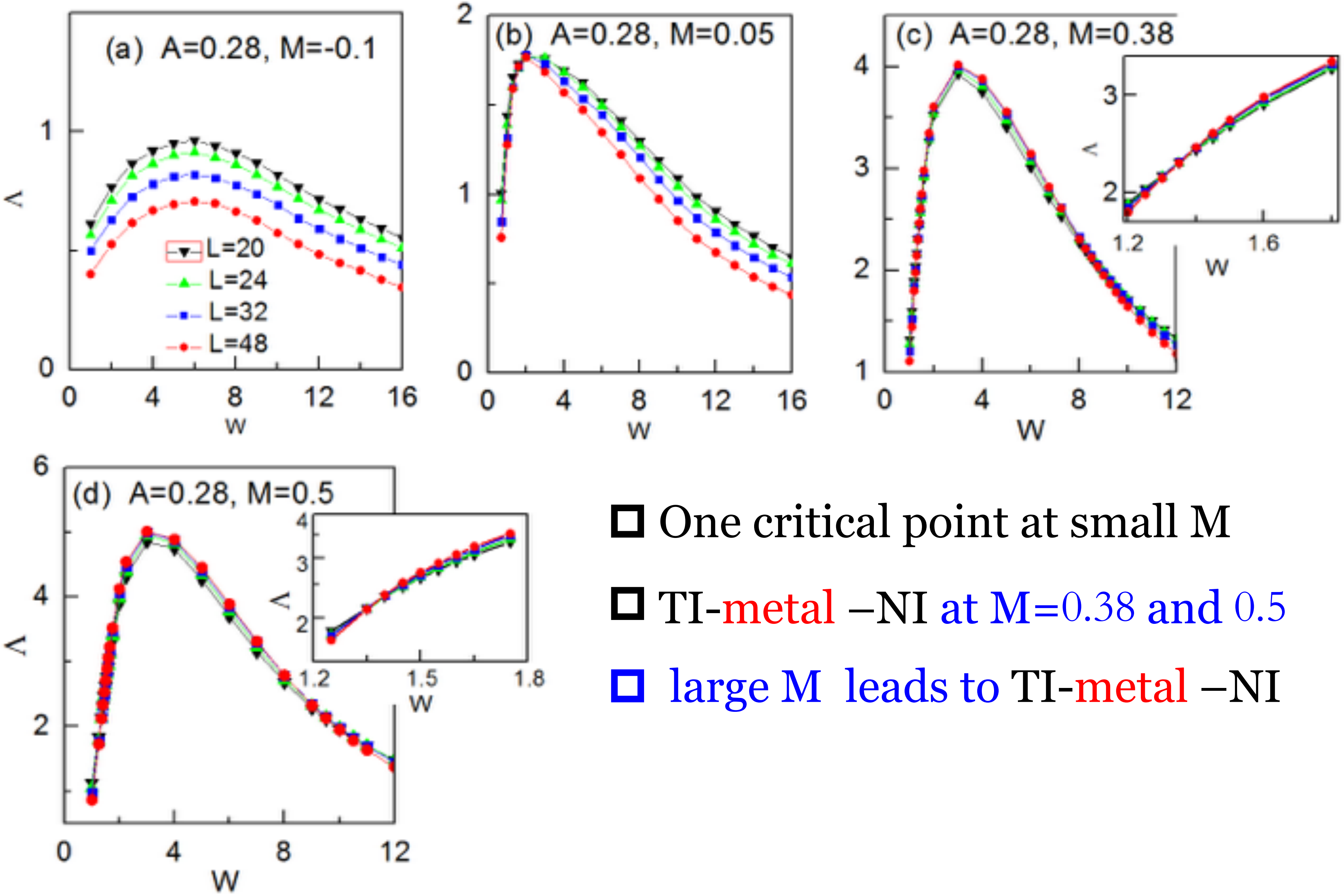
$$H_0(k) = A(k_x\tau_x + k_y\tau_y) + (M - Bk^2)\tau_z + V_{dis}\tau_0$$

Investigations in four possible ways:

- **(I)Scaling localization length**
- **(II)Energy level statistics**
- **(III)Participation ratio**
- **(IV)Intrinsic conductance**

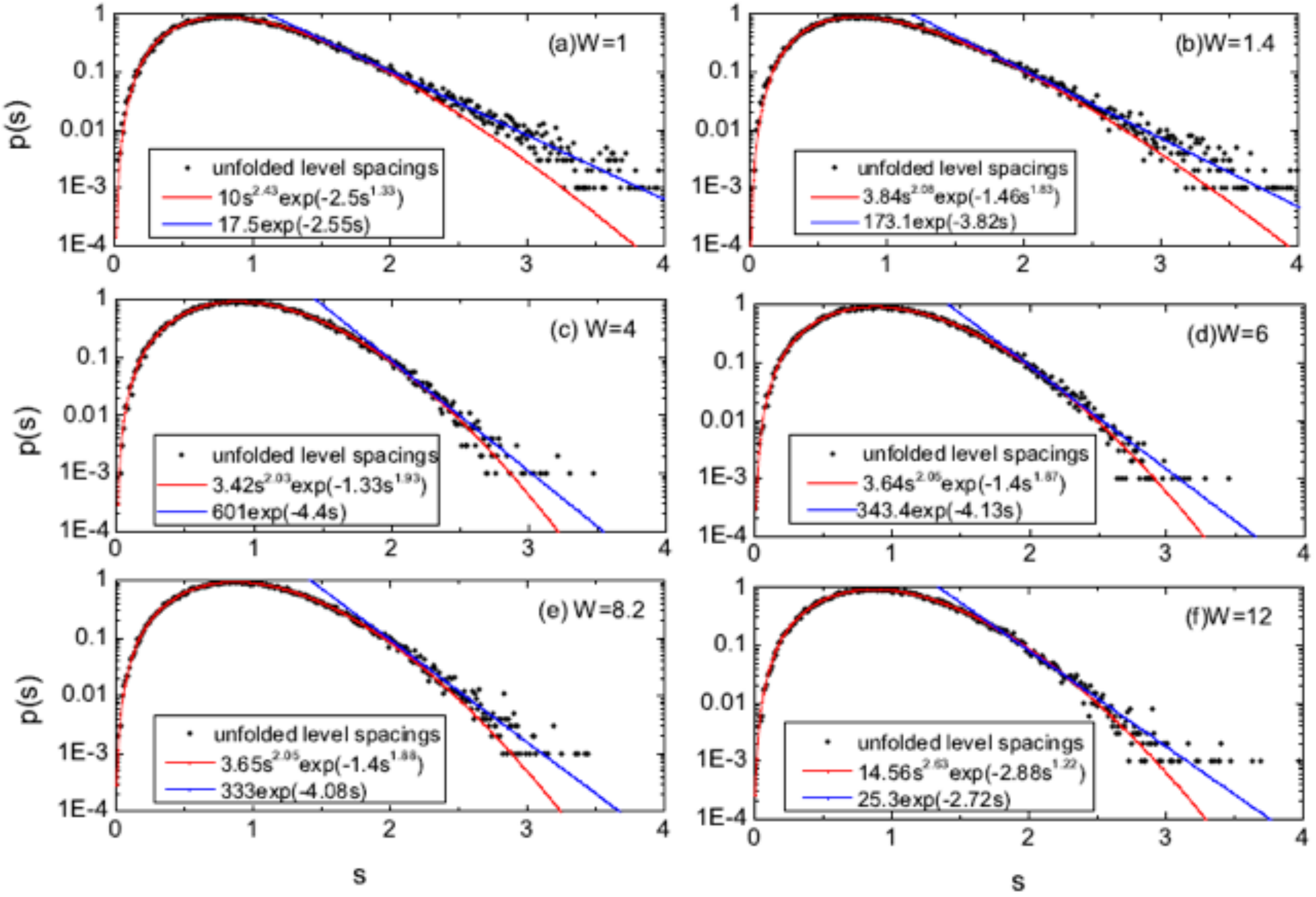


## (I) Localization length scaling at different masses M




- One critical point at small M
- TI-metal –NI at M=0.38 and 0.5
- large M leads to TI-metal –NI

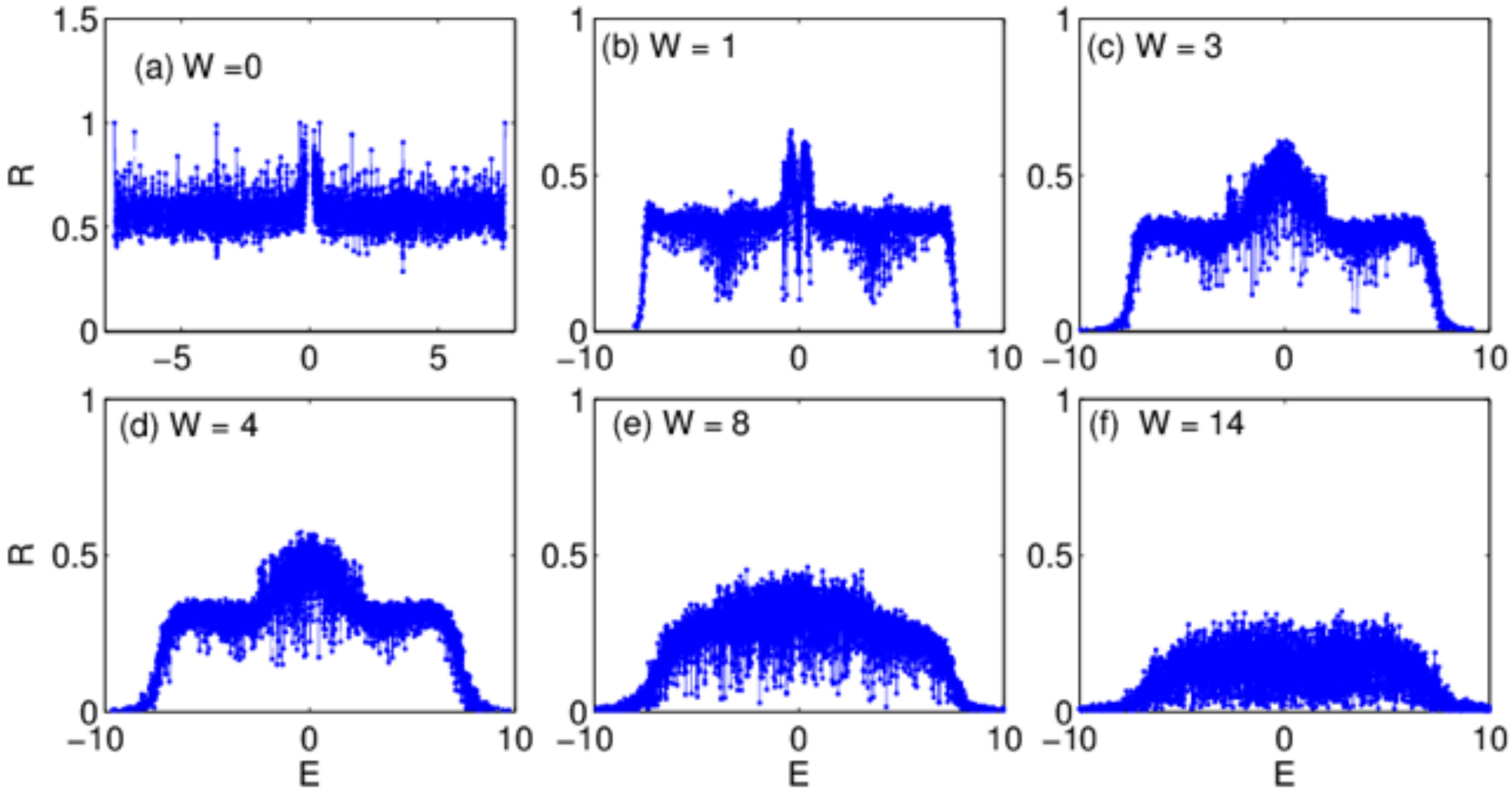
# (II) Energy level statistics (ELS) at A=0.28, M=0.38 and EF=0




□ Supporting the existence of extended states

## (III) Participation ratio (PR)

PR: $R = \sum_{i=1}^{N} |a_i|^4 / (N \sum_{i=1}^{N} |a_i|^2)^2$



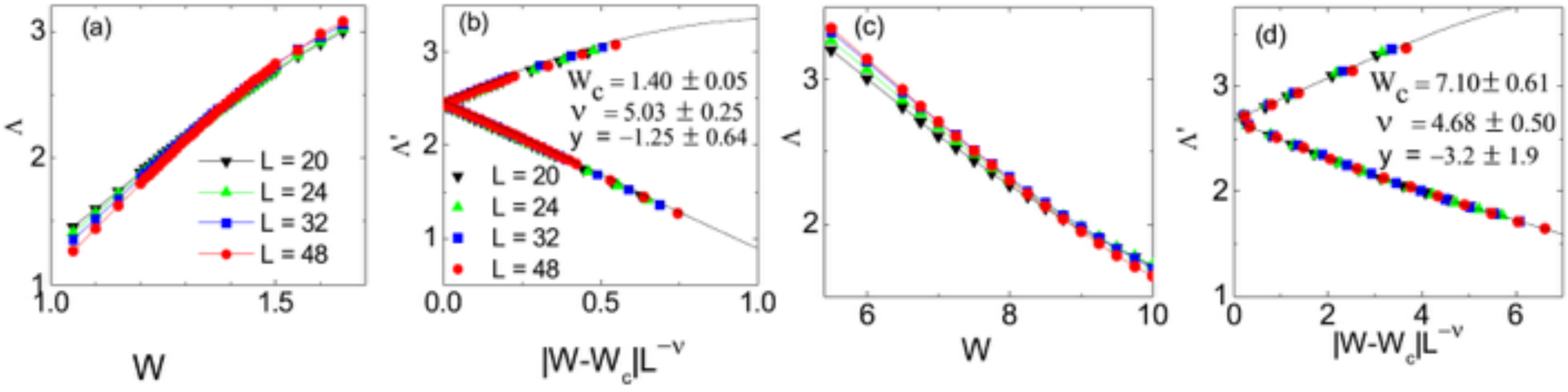

- The extended states come into the gap with increasing disorder strength.

## ◻ One parameter scaling behaviour

$$\Lambda(W,L) = \Lambda_c + \sum_{n=1}^{4} a_n (W - W_c)^n L^{-n/\nu} + b_0 L^y$$

$$\Lambda'(W,L) = \Lambda(W,L) - b_0 L^y$$

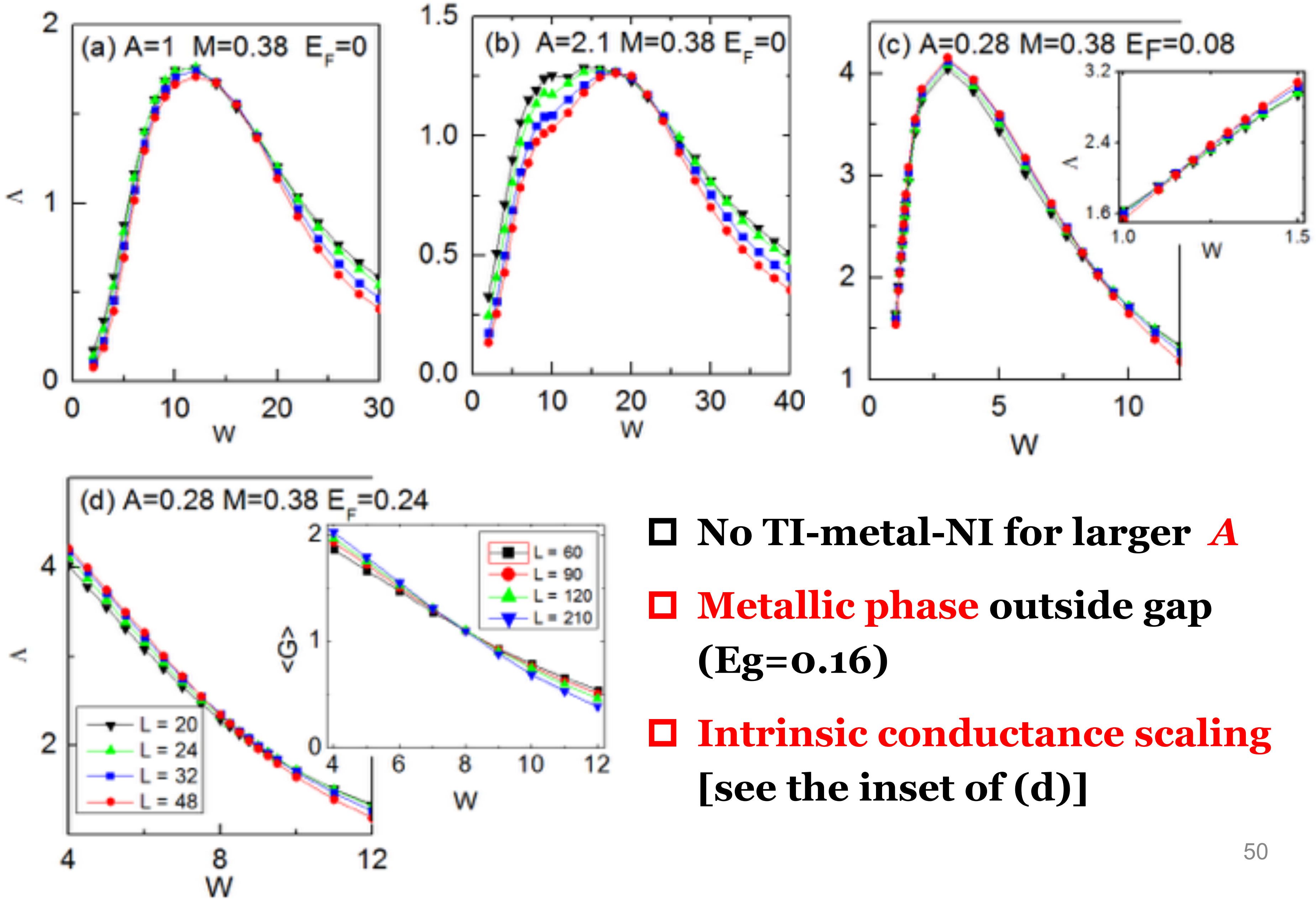
A=0.28, M=0.38, $E_F$=0




$$\chi^2 = \sum_{n=1}^{N} (\Lambda_n - \Lambda(W_n, L_n))^2/\sigma_n^2$$

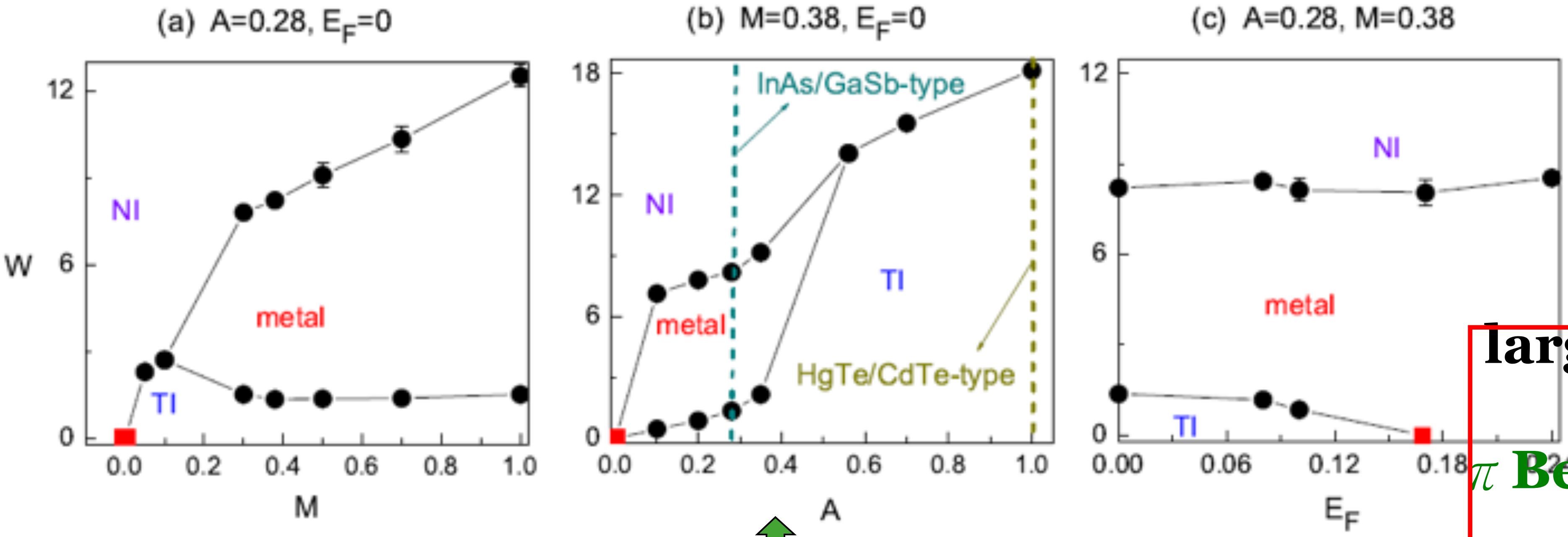
$\chi^2/N$ is 3.3 and 3.5 for (b) and (d) respectively.

# AT with different electron-hole hybrid strength and Fermi energy




- No TI-metal-NI for larger *A*
- **Metallic phase** outside gap (Eg=0.16)
- **Intrinsic conductance scaling** [see the inset of (d)]

## Phase diagrams




- large $M$ and small $A$
- $\pi$ Berry phase or WAL

$$\beta = \frac{d \ln g}{d \ln L} = d - 2 - \frac{c}{g} > 0$$

**TI-Metal-NI**

## □ Berry phase

$$\gamma(E_F) = \left(1 - \frac{(M - Bk_F^2)}{\sqrt{A^2k_F^2 + (M - Bk_F^2)^2}}\right)\pi$$

- **When $\gamma \to (0 \to \pi \to 2\pi)$, weak localization (WL)—>WAL—>WL**
  - ◆ **Beta-function is positive for WAL in 2D**
  - ■ **InAs/GaSb-type, $M/B \gg A^2/B^2$, $\gamma(E_g) \approx \pi$ -> WAL**
  - ■ **HgTe/CdTe-type, $M/B < A^2/B^2$, $\gamma(E_g) = 0$ -> WL**

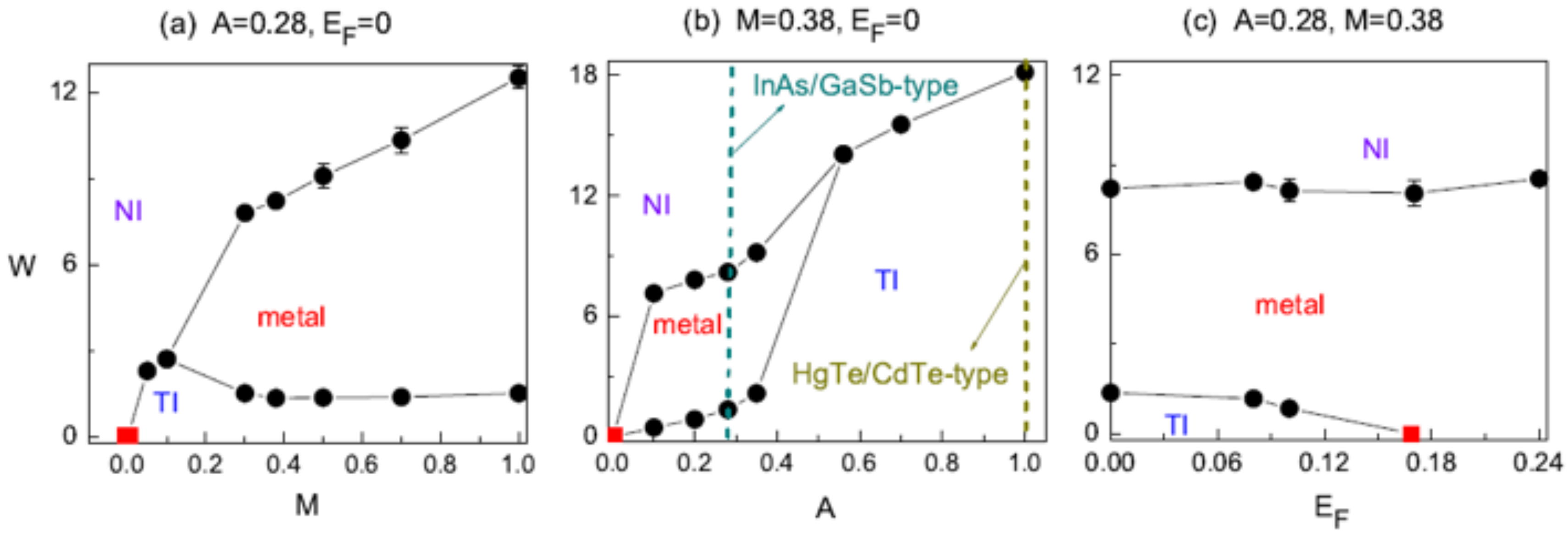
# Conclusion on Tunable Anderson transition in QSH insulator

➢Anderson transition is tunable by model parameters in the BHZ model.

➢We find a possible metallic phase in 2D unitary class.

❑ **localization length scaling, the conductance scaling, energy level statistics and participation ratio.**

➢Our results are interpreted by the Berry phase or WAL.



# Outline

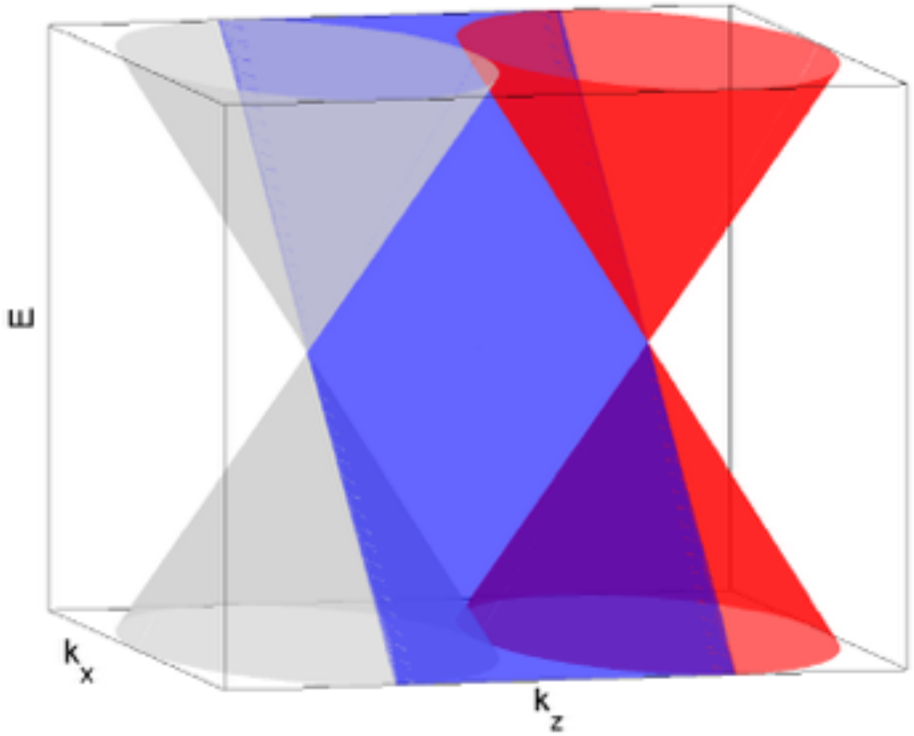
- ➢ Introduction
  - ❑ From quantum Hall systems to topological insulators
- ➢ Dephasing effects in topological insulators
  - ❑ Edge states of 2D quantum spin Hall insulators
  - ❑ Surface states of 3D topological insulators
- ➢ Disorder effects in topological insulators
  - ❑ Tunable Anderson transition in 2D quantum spin Hall insulators
  - ❑ Multiple Anderson transition in Weyl semimetals

# Hamiltonian and power spectrums

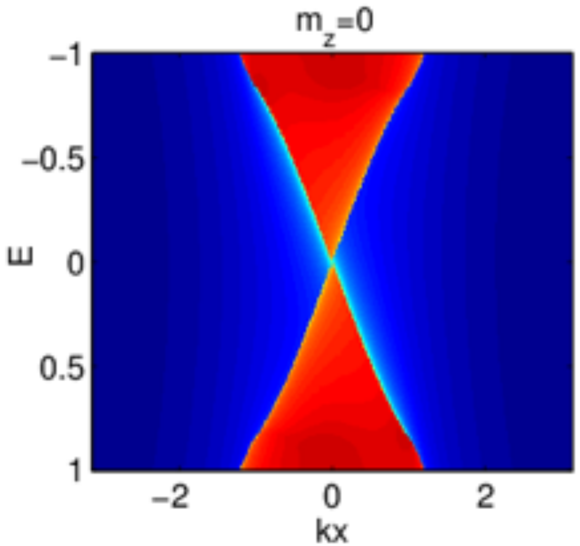
## ◻ Hamiltonian

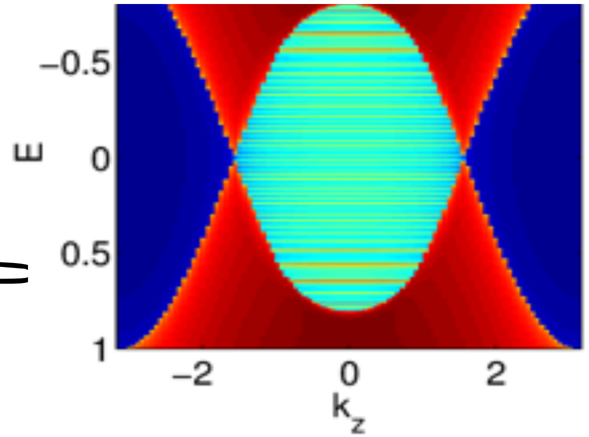
$$H_0 = M_k \sigma_z + t_x \sin k_x \sigma_x + t_y \sin k_y \sigma_y$$

**where** $M_k = m_z - t_z \cos k_z + m_0(2 - \cos k_x - \cos k_y)$




parameters: $t_x = t_y = t_z = 1,$
$m_0 = 2.1, m_z = 0$



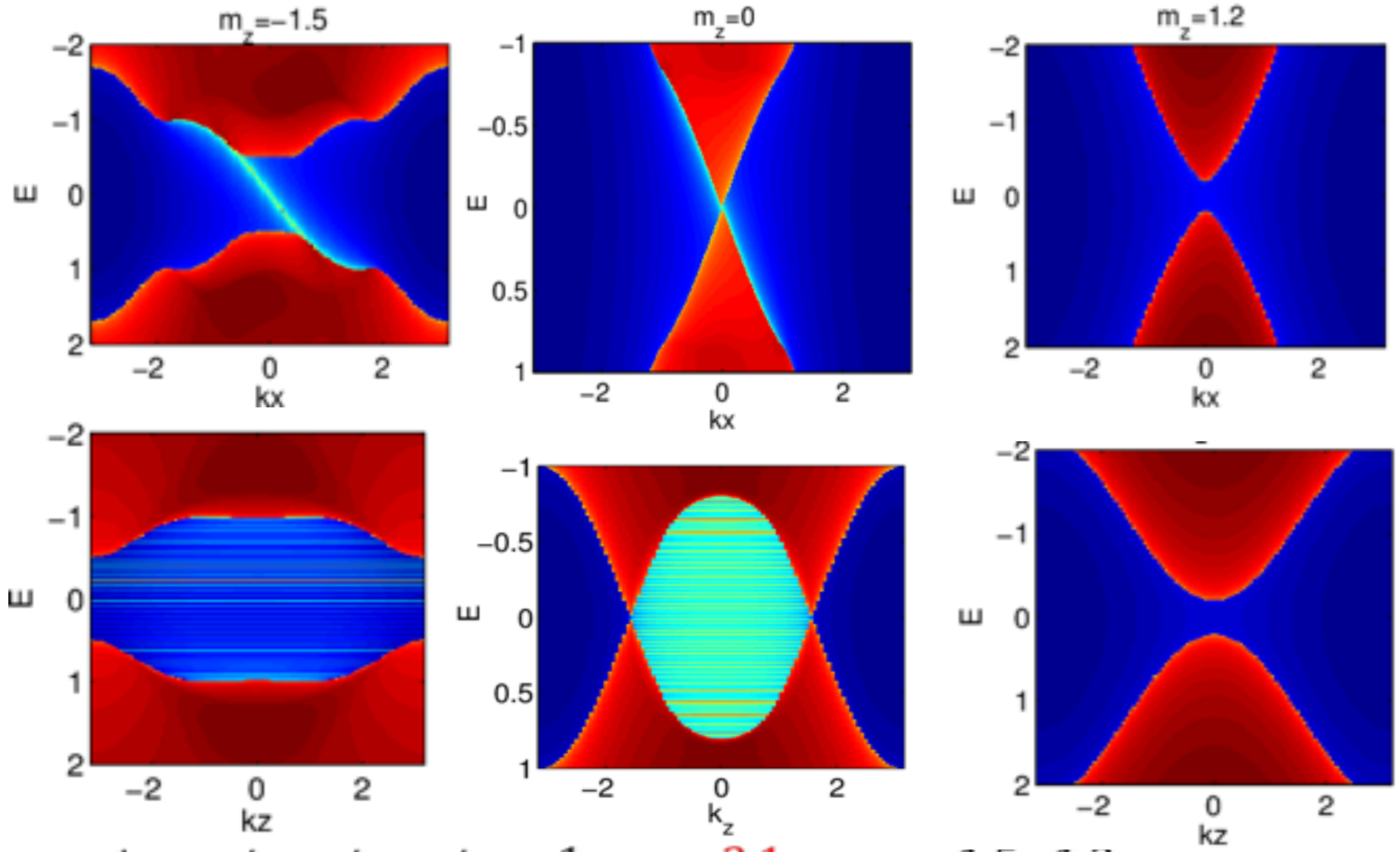



## Hamiltonian and power spectrums (cont'd)

### □ Hamiltonian

$$H_0 = M_k\sigma_z + t_x \sin k_x\sigma_x + t_y \sin k_y\sigma_y$$

**where** $M_k = m_z - t_z \cos k_z + m_0(2 - \cos k_x - \cos k_y)$



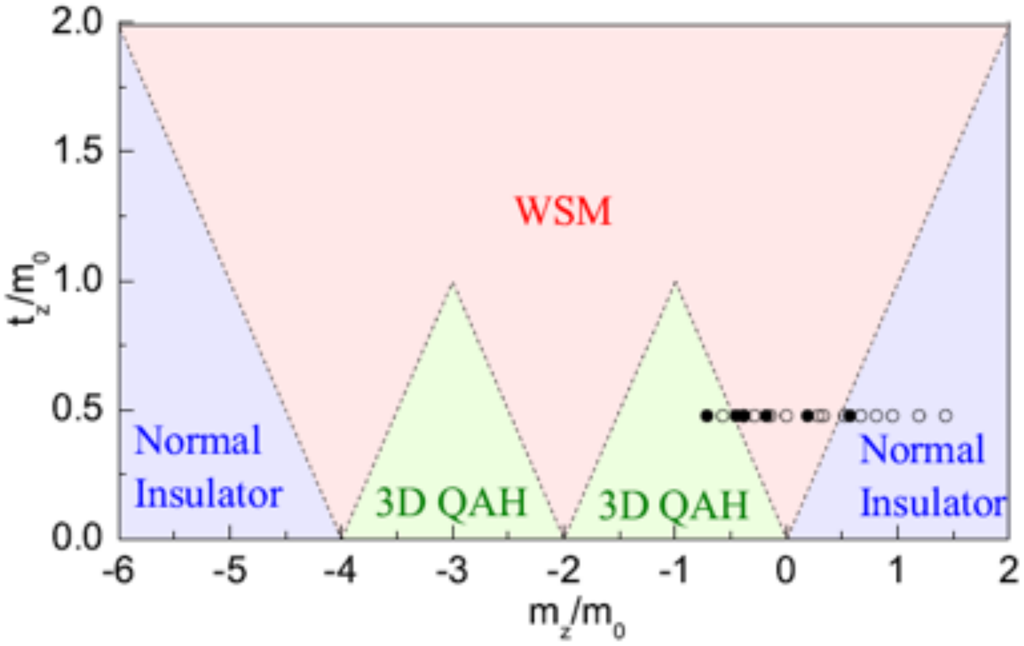

paramters: $tx = ty = tz = 1, m_0 = 2.1, m_z = -1.5\sim1.2$

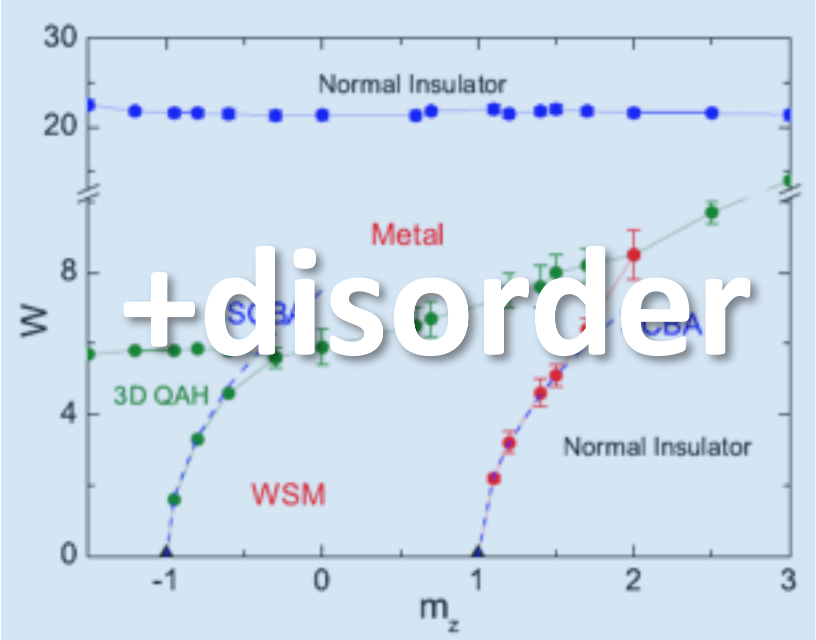
# Phase diagram in clean limit + disorder

## □ Clean Hamiltonian and phase diagram

$$H_0 = M_k\sigma_z + t_x \sin k_x\sigma_x + t_y \sin k_y\sigma_y$$

**where** $M_k = m_z - t_z \cos k_z + m_0(2 - \cos k_x - \cos k_y)$



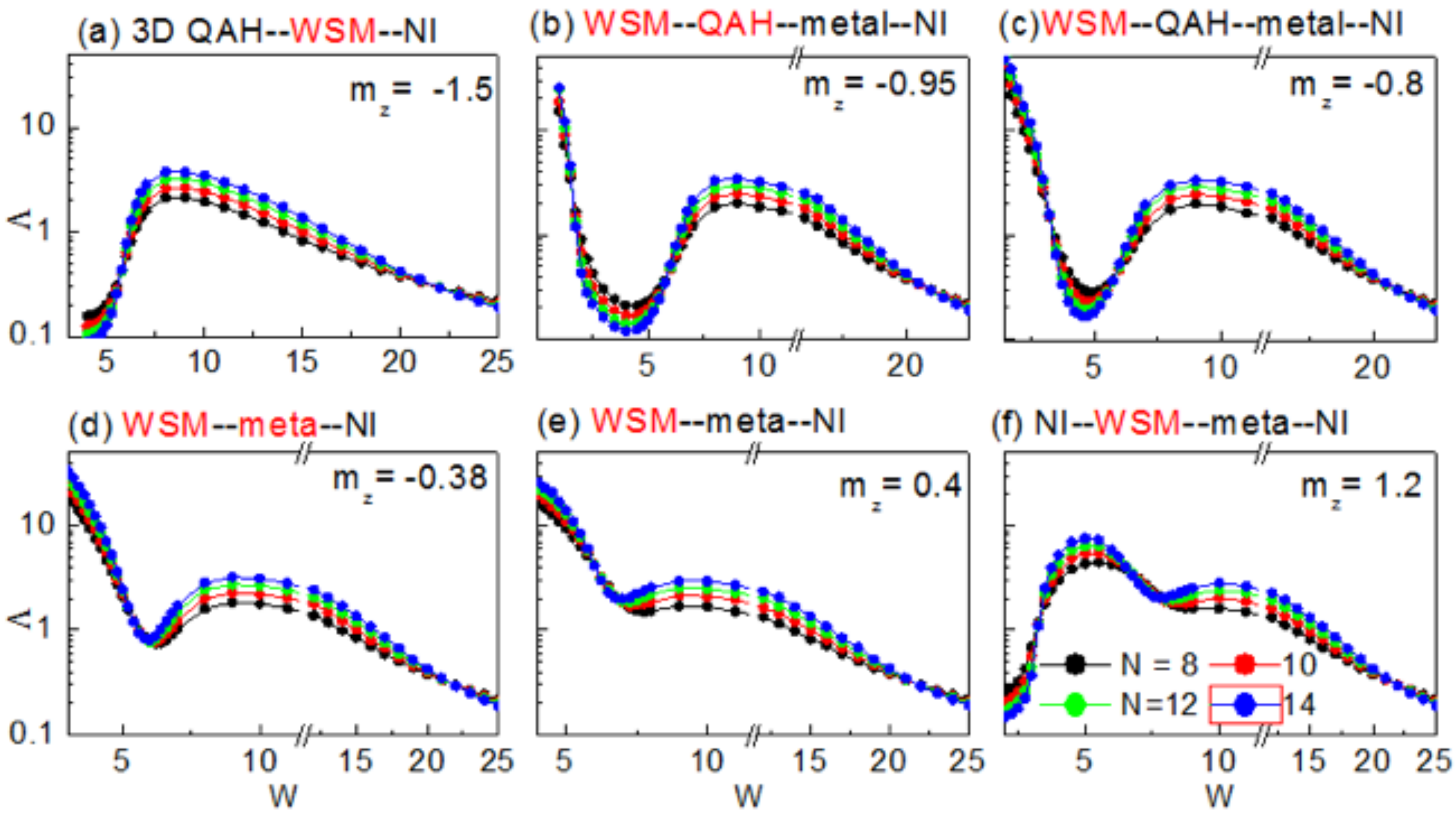

paramters: $tx = ty = tz = 1,$
$m_0 = 2.1, m_z = -1.5 \sim 1.2$

## □ Disorder Hamiltonian

$$H = H_0 + \begin{pmatrix} V_1(\mathbf{r}) & 0 \\ 0 & V_2(\mathbf{r}) \end{pmatrix} \qquad V_{1,2}(\mathbf{r}) \in [-W/2, W/2]$$

# Multiple Anderson transition

- **Renormalized localization length** $\Lambda$ **v.s. disorder strength W**

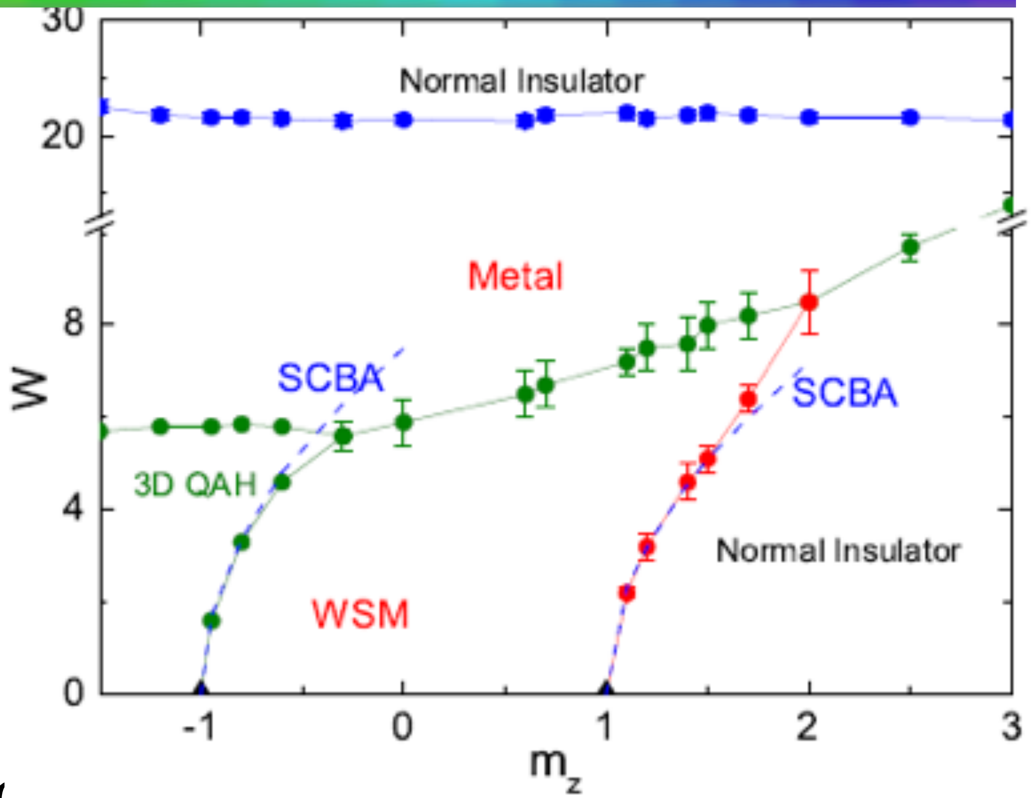



- **WSM—QAH, NI—WSM : SCBA**
- **WSM—metal: bulk states**

# Phase diagram and multiple Anderson transition

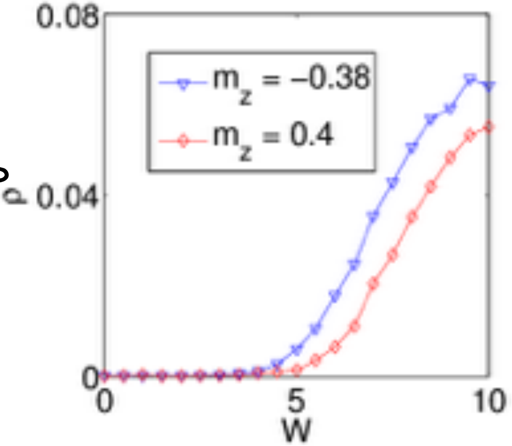
➢ multiple Anderson transition

- ☐ QAH—metal—NI
- ☐ WSM—QAH—metal—NI
- ☐ WSM—metal—NI
- ☐ NI—WSM—metal—NI
- ☐ NI—meta—NI




➢ Exotic phase transitions

- ◆ WSM—QAH： nontrival gap (
- ◆ WSM—metal: emergent bulk states
- ◆ NI—WSM : gap closing (SCBA)



# Phase diagram and multiple Anderson transition

## ◻ Clean Hamiltonian

$$H_0 = M_k \sigma_z + t_x \sin k_x \sigma_x + t_y \sin k_y \sigma_y$$

**where** $M_k = m_z - t_z \cos k_z + m_0(2 - \cos k_x - \cos k_y)$

## ◻ Hall conductance of WSMs

$$\sigma_{xy} = \sum_{k_z \in BZ} \sigma_{xy}^{2D}$$

$$\mathrm{kz} = 0, \pm\frac{2\pi}{N_z}, \pm\frac{2\pi}{N_z} * 2, \pm\frac{2\pi}{N_z} * 3, \ldots, \pi$$

$$\sigma_{xy}^{2D}(k_z) = \Theta(k_0 - |k_z|)e^2/h,$$

**Weyl nodes $\pm k_0$ with $k_0 = arccos(m_z/t_z)$**

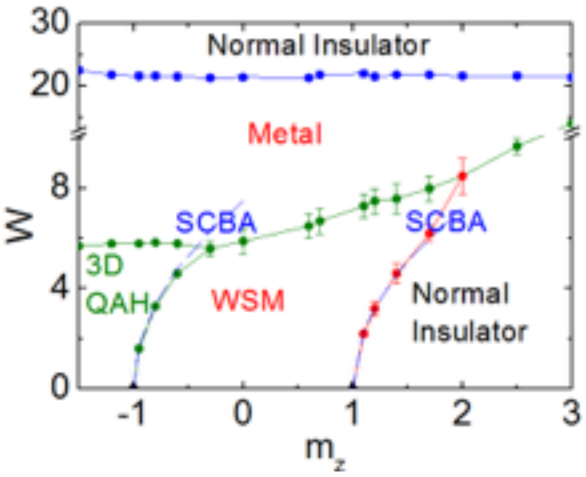
**For example** Lz = 8

| | | |
|---|---|---|
| $1 < m_z$ | NI | = 0 |
| $-1 < m_z < 1$ | WSM | = 1, 3, 5, 7 |
| $-1.5 < m_z < -1$ | 3D QAH | = 8 |
| $1 < m_z$ | NI | = 0 |
| $-1 < m_z < 1$ | WSM | = 1, 3, 5, 7 |
| $-1.5 < m_z < -1$ | 3D QAH | = 8 |
| $1 < m_z$ | NI | = 0 |
| $-1 < m_z < 1$ | WSM | = 1, 3, 5, 7 |
| $-1.5 < m_z < -1$ | 3D QAH | = 8 |

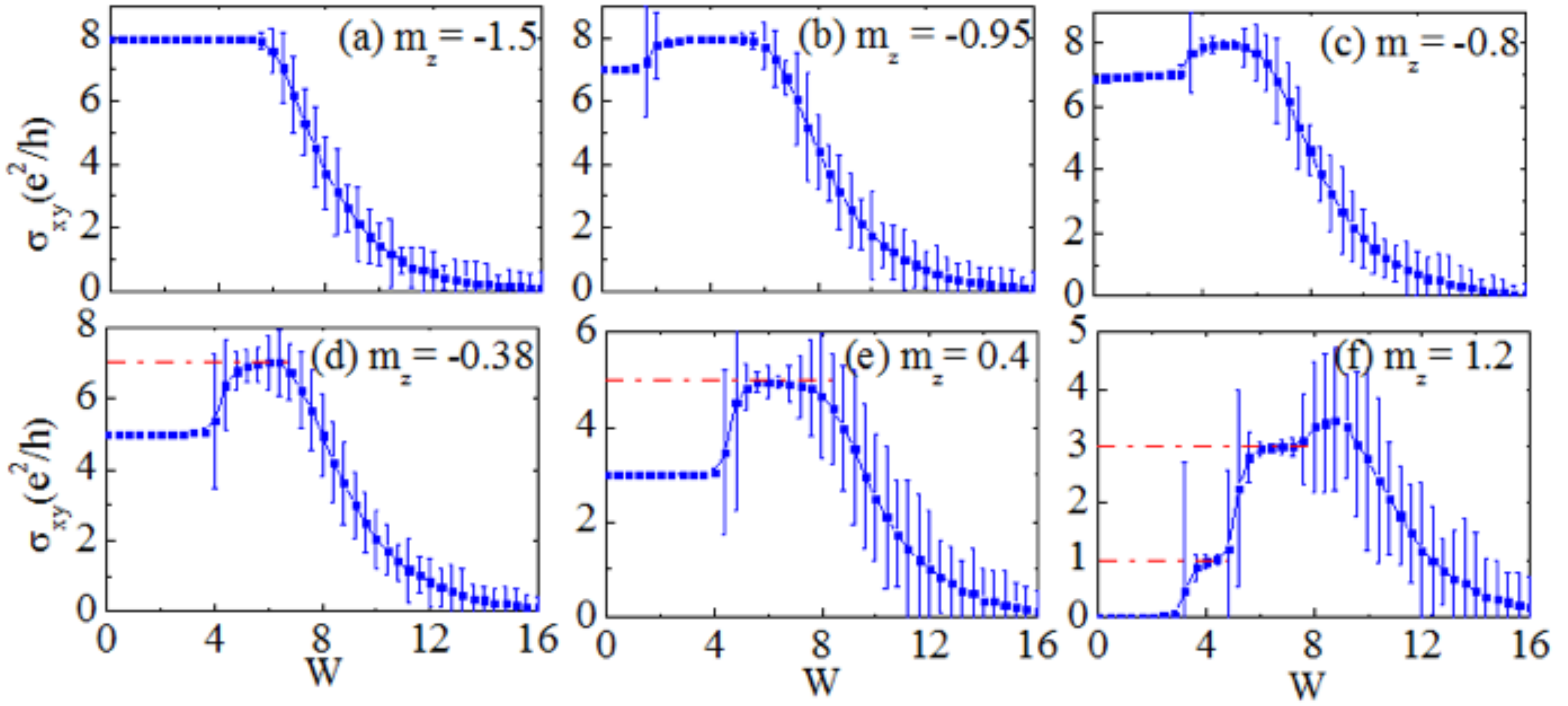
**NI** $\sigma_{xy}$ = 0

**WSM** $\sigma_{xy}$ = 1, 3, 5, 7

**3D QAH** $\sigma_{xy}$ = 8



# Surface states and Hall conductance in x-y plane

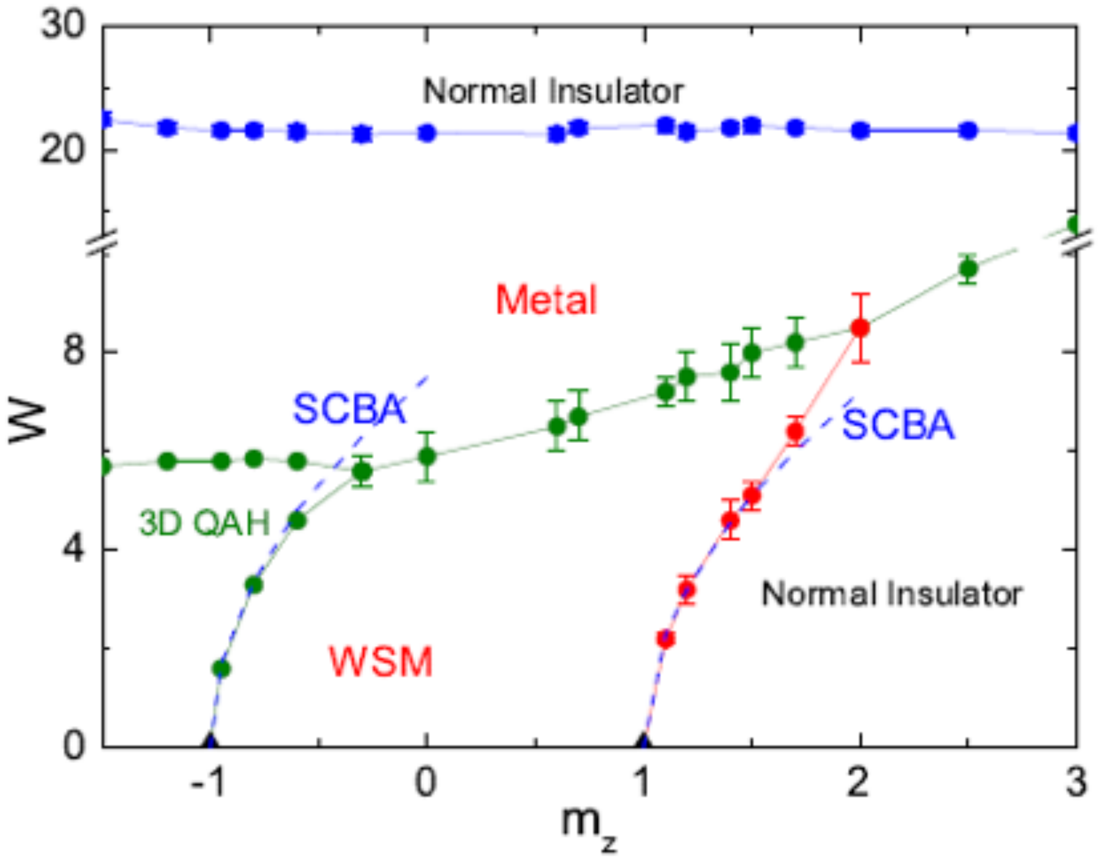



- **WSM--QAH:** $\sigma_{xy} = (7 \rightarrow 8)\frac{e^2}{h}$ **reach BZ boundary**
- **WSM--metal: quantized to non-quantized Hall conductance**
- **NI--WSM:** $\sigma_{xy} = (0 \rightarrow 1)\frac{e^2}{h}$

# Conclusion on Anderson transitions in WSMs

➢ multiple Anderson transitions

- **QAH—metal—NI**
- **WSM—QAH—metal—NI**
- **WSM—metal—NI**
- **NI—WSM—metal—NI**
- **NI—meta—NI**




◆ **WSM--metal**

- **two extended states**
- **Hall conductance**

◆ **WSM--3D QAH**

- **SCBA and Hall conductance**

# Related Publications:


H. Jiang et al., PRL, 103, 036803 (2009);

Q.F. Sun et al., PRL, 104,066805 (2010) ;

S. L Yu et al., PRL, 107, 010401 (2011);

H. Jiang et al., PRL, 112, 176601 (2014) ;

H. W. Liu et al., PRL, 113, 046805 (2014);

H. Jiang et al., PRB, 80, 165316 (2009);

H. Zhang et al., PRB, 83, 115402 (2011);

L. Wang et al., PRB, 84, 205116 (2011);

X. L. Liu et al., PRB, 83, 125105 (2011);

J. T. Song et al., PRB, 85, 195125 (2012);

D. W. Xu et al., PRB, 85, 195140 (2012);

L. Wang et al., PRB, 85, 235135 (2012);

X. L. Liu et al., PRB, 85, 235459 (2012);